Expert Journal of Economics. Volume 14, pp. 1-16, 2026

ISSN 2359-7704
Economics.ExpertJournals.com

Portfolio Optimization of Financial Securities Under the Mean-Semivariance Behavioral Hypothesis

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This paper examines financial portfolio optimization by applying the Post-Modern Portfolio Theory (PMPT) to the Romanian capital market. The study is based on a sample of five representative companies included in the BET Index of the Bucharest Stock Exchange: Nuclearelectrica, Hidroelectrica, BRD – Groupe Société Générale, Digi Communications N.V., and Fondul Proprietatea. These companies were selected based on their high liquidity and representation of different economic sectors, both of which are essential for portfolio diversification and for ensuring continuous and comparable data series. The analysis employs a sample of 252 daily observations covering the period from 18 December 2024 to 30 December 2025.
The research applies an extension of the Markowitz model based on the mean–semivariance behavioral hypothesis, focusing on downside risk rather than the traditional mean–variance framework. Accordingly, downside deviation, semivariance, and semicovariance were calculated for the selected financial assets in order to construct the semivariance-semicovariance matrix. The optimal asset weights were determined using the Lagrange multiplier method, resulting in the construction of two portfolio types: the minimum-risk portfolio and the efficient portfolio corresponding to a target expected return. Portfolio performance was also evaluated using the Sortino ratio.
The results indicate that portfolio composition is directly influenced by the risk–return trade-off and confirm the applicability of Post-Modern Portfolio Theory to the Romanian capital market. Furthermore, the analysis highlights the importance of diversification and the role of defensive assets in reducing investment risk.

Keywords: *Post-Modern Portfolio Theory (PMPT), Mean-Semivariance, Portfolio Optimization, Downside Risk, Romanian Capital Market*

JEL Classification: *G11, G17*

1. Introduction

Capital markets constitute an essential component of the global economic system, providing not only vital sources of financing for corporations but also investment opportunities accessible to both companies and individuals across all segments of society. These markets are not merely trading mechanisms but also

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Article History:
Received 25 May 2026 | Accepted 22 July 2026 | Available online 30 July 2026

Cite Reference:
Feldeorean, S. and Brătian, V.R., 2026. Portfolio Optimization of Financial Securities Under the Mean-Semivariance Behavioral Hypothesis. *Expert Journal of Economics, 14*(1), pp.1-16.

barometers of economic health, reflecting expectations and responses to macroeconomic and political developments. From the perspective of portfolio diversification and optimization, capital markets have been extensively examined in the academic literature through a variety of economic theories and models. This paper seeks to explore an essential aspect: the structure of a minimum-risk portfolio and the optimal structure of a portfolio corresponding to a target return set by the investor.

For this analysis, five companies were selected from the twenty constituents of the BET Index: Nuclearelectrica, representing the energy sector; BRD – Groupe Société Générale, representing the banking sector; Digi Communications N.V., a major telecommunications company; Fondul Proprietatea, one of the largest closed-end investment funds listed on the Bucharest Stock Exchange; and Hidroelectrica, an energy company specializing in hydroelectric power generation. The selection of these stocks contributes to portfolio diversification and enables the application of the principles of PMPT to identify both the minimum-risk portfolio and the optimal portfolio corresponding to a predetermined target return.

The paper is organized into three main sections. The first section presents the theoretical framework, including a review of the relevant literature and the key concepts underlying the research, namely the concept of a securities portfolio, the differences between Modern Portfolio Theory and Post-Modern Portfolio Theory, the Markowitz efficient frontier, the minimum-risk portfolio, and the Sortino ratio. The second section describes the research methodology, while the final section presents the empirical findings, discusses the results, and provides the concluding remarks.

The decision to investigate Post-Modern Portfolio Theory is motivated by its significant contribution to financial portfolio optimization through the integration of expected return with the level of risk undertaken. This theory has influenced the development of economic models and investment strategies by emphasizing downside risk rather than total volatility. Nevertheless, Post-Modern Portfolio Theory remains subject to debate, as it is often criticized for the challenges associated with empirical testing compared with Modern Portfolio Theory. This issue highlights a fundamental challenge in economic research: the continuous need to assess the validity and applicability of theoretical frameworks.

Another motivation underlying this study is the practical value of mastering these portfolio optimization techniques for a future finance professional. In a corporate environment, where responsibilities may include managing financial market investments and overseeing investment budgets, a thorough understanding of these methods can support more effective investment decision-making and improve the risk–return trade-off. Accordingly, this paper focuses on analyzing the structure of financial portfolios within the framework of Post-Modern Portfolio Theory.

2. Theoretical framework

This study is grounded in the academic literature on Modern Portfolio Theory and its extension, Post-Modern Portfolio Theory. To develop a comprehensive understanding of portfolio optimization mechanisms and semivariance-based models, both seminal contributions and recent empirical studies in the field of financial portfolio management were examined.

The literature review was not limited to studies directly addressing the determination of the minimum-risk portfolio or the optimization of portfolios for a specified target return. It also included theoretical and conceptual works that provide the mathematical and behavioral foundations of the models employed in this research. Accordingly, the review covers key concepts such as portfolio diversification, the efficient frontier, downside risk, semivariance, the Sortino ratio, and loss aversion.

Accordingly, the study draws upon seminal works such as those of Harry Markowitz (1959) on Modern Portfolio Theory, the contributions of Frank Sortino (1994; 1991) concerning downside risk and risk-adjusted performance, and the research in the field of behavioral finance conducted by Daniel Kahneman and Amos Tversky (1979).

In addition, recent empirical studies on the applicability of financial portfolio optimization models and the evaluation of investment performance were examined. (Şerban et al., 2025; Brătian, 2024; Venkataraman, 2023; Harville, 2023; Kahraman & Somuncu, 2023; Bratian, 2018)

2.1. The Concept of a Securities Portfolio

A securities portfolio refers to the collection of financial securities held simultaneously by an investor, with each asset representing a specific proportion of the total invested capital. The primary objectives of a securities portfolio are to generate returns and to manage investment risk through diversification (Brătian et al., 2016).

2.2. Modern Portfolio Theory (MPT)

In 1952, Harry Markowitz laid the foundations of Modern Portfolio Theory (MPT) through his seminal paper *Portfolio Selection*. Subsequently, in 1964, William F. Sharpe extended the theoretical framework of capital markets by developing the Capital Asset Pricing Model (CAPM).

Together, these contributions established the theoretical foundation of Modern Portfolio Theory, enabling investors to construct and select portfolios based on the relationship between expected return and investment risk. Within this framework, investors seek either to maximize expected return for a given level of risk or to minimize risk for a specified level of expected return. The resulting set of optimal portfolios is graphically represented by the efficient frontier (Mangram, 2013; Brătian et al., 2016).

It is important to note that Markowitz's Portfolio Selection Theory is considered a normative theory, as it prescribes how investors should construct their portfolios. In contrast, the Capital Asset Pricing Model (CAPM) developed by Sharpe is regarded as a positive theory, as it seeks to explain how asset returns are determined in capital markets (Mangram, 2013).

The portfolio selection model developed by Harry Markowitz in his seminal paper Portfolio Selection is based on a set of assumptions concerning investor behavior and the functioning of financial markets. The main assumptions are as follows: (1)Investors are rational and seek to maximize expected returns while minimizing risk; (2) Investors are willing to accept higher levels of risk only if they are compensated with higher expected returns; (3) Investors have access to all relevant information necessary for making investment decisions; (4) Investors can borrow or lend funds at the risk-free rate; (5) Financial markets operate efficiently, and asset prices fully reflect the available information; (6) There are no taxes or transaction costs that influence investment decisions; (7)Financial assets can be combined into diversified portfolios, thereby reducing the overall investment risk (Mangram, 2013).

2.3. Post-Modern Portfolio Theory (PMPT)

Following the development of Modern Portfolio Theory (MPT) by Harry Markowitz in 1952, subsequent research identified several limitations of the model, particularly with respect to the measurement of investment risk (Tsai & Wang, 2012). In this context, Frank A. Sortino revisited the concept of semivariance, originally introduced by Markowitz (1959), and further developed it into a more practical and comprehensive theoretical framework.

Thus, in downside risk, Frank A. Sortino established the foundations of an alternative approach to portfolio theory focused on the assessment of downside risk (Sortino & Van der Meer, 1991). These ideas were subsequently expanded and systematized within what the academic literature has come to define as Post -Modern Portfolio Theory (PMPT), developed by Brian M. Rom and Kathleen Ferguson.

From a behavioral perspective, PMPT is based on the premise that investors exhibit loss aversion, assigning greater importance to negative returns than to favorable fluctuations. This approach is consistent with the findings of behavioral finance, which suggest that investors do not perceive gains and losses symmetrically (Kahneman & Tversky, 1979).

The principal difference between MPT and PMPT lies in the definition of risk. While MPT measures risk using the standard deviation of returns, PMPT focuses primarily on negative return deviations, namely downside risk, which is quantified through downside deviation (semideviation) (Jianu et al., 2013).

Another important distinction is the introduction of the minimum acceptable return as a benchmark in place of the average return. In addition, the Sharpe ratio is replaced by the Sortino ratio, which measures risk-adjusted performance by considering only downside risk. However, the application of PMPT involves a higher degree of complexity in estimating investment risk (Brătian, 2024).

In this context, Table 1 presents a concise overview of the key differences between MPT and PMPT, emphasizing the distinct approaches adopted by the two frameworks in terms of risk assessment, optimization criteria, and performance measurement.

Table 1. Differences between MPT and PMPT

Theory / Characteristic	MPT	PMPT
Behavioral assumption	Risk aversion	Loss aversion
Optimization criterion	Return–Risk	Return–Downside Risk
Risk measure	Variance	Semivariance

Performance measure	Sharpe ratio	Sortino ratio

Source: (Șerban, et al., 2025)

Overall, researchers largely agree on the benefits of diversification and the importance of optimizing the risk–return trade-off. However, one of the principal criticisms of Modern Portfolio Theory MPT is that it treats both positive and negative fluctuations in returns as sources of risk. In contrast, Post-Modern Portfolio Theory (PMPT) considers only returns falling below a threshold chosen by the investor, most commonly the risk-free rate to represent investment risk.

Nevertheless, some academics and practitioners argue that the distinction between MPT and PMPT is not sufficiently significant to justify the additional computational complexity required by the latter. From this perspective, diversification remains the most effective approach to portfolio optimization (Cheng & Wolverton, 2001; Mangram, 2013).

2.4. The Markowitz Efficient Frontier

The efficient frontier consists of the set of all efficient portfolios. An efficient portfolio is an optimal combination of risk and return for which no other portfolio offers a higher expected return at the same level of risk or a lower level of risk for the same expected return. (Harville, 2023; Mangram, 2013).

2.5. The Minimum-Risk Portfolio

The minimum-risk portfolio is an efficient portfolio and represents the starting point of the efficient frontier. It consists of the combination of assets whose weights are determined so as to minimize the overall portfolio risk. Consequently, among all feasible portfolio allocations, it offers the lowest possible level of risk for the investor. Such a portfolio is typically preferred by risk-averse investors, who seek to minimize exposure to investment risk (Brătian et al., 2016; Stancu & Stancu, 2012).

2.6. Sortino Ratio

Since the Sharpe ratio measures risk across the entire return distribution by using the standard deviation of returns, it is not fully appropriate for the purposes of this study. Instead, the Sortino ratio is employed as the primary performance measure, as it considers only the downside deviations of returns (Sortino & Price, 1994).

3. Methodology

The methodology employed in this study is presented below, as follows:

Let

$$r_{i,t} = ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \tag{1}$$

where:

$r_{i,t}$ = the daily logarithmic return of security *i* at time *t*

$P_{i,t}/P_{i,t-1}$ = the prices of security *i* at times *t* and *t – 1*, respectively.

To capture exclusively the risk associated with losses, the downside deviations of asset returns are first calculated. These downside deviations are then used to compute the semivariance and semicovariance of the assets, which form the semivariance–semicovariance matrix. (Estrada, 2008; Venkataraman, 2023).

We opt for the daily logarithmic returns ($r_{i,t}$) of the assets in the portfolio because they are approximately equal to simple (arithmetic, $R_{i,t}$ returns). Following a Taylor series expansion, we obtain $r_{i,t} = \ln\left(1 + R_{i,t}\right) \approx R_{i,t}$, as $R_{i,t} \rightarrow 0$. Furthermore, logarithmic data series eliminate the extreme asymmetry caused by the compounding effect. The distribution becomes more stable (closer to a normal distribution, although it retains fat tails), which allows the downside risk (downside deviation) proposed by Estrada to capture the real risk of loss much more accurately.

Downside deviation:

$$\sigma_{i(d)} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} d_{i,t}^{2}} \tag{2}$$

where: MAR = minimum acceptable return; $d_{i,t} = \min(r_{i,t} - MAR, 0)$; $\sigma_{i(d)}$ = downside deviation; T = total number of daily observations in the sample.

Semivariance of asset i:

$$\sigma^2_{i(d)} = \frac{1}{T}\sum_{t=1}^{T} d^2_{i,t} \tag{3}$$

Semicovariance between assets i and j:

$$\sigma_{ij(d)} = \frac{1}{T}\sum_{t=1}^{T} d_{i,t} \cdot d_{j,t} \tag{4}$$

The mathematical model is based on two key components: expected return and risk, measured by semivariance. The optimization is subject to the budget constraint and the non-negativity constraint on the portfolio weights, as follows:

$$\mu_p = \sum_{i=1}^{n} \omega_i \cdot \mu_i \tag{5}$$

$$\sigma^2_{p(d)} = \sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i \omega_j \sigma_{ij(d)} \tag{6}$$

$$\sum_{i=1}^{n} \omega_i = 1, \text{ as } \omega_i \geq 0 \tag{7}$$

where: μ_p = expected portfolio return; μ_i = expected return of asset *i*; ω_i = weight of asset *i* in the portfolio; ${\sigma^2}_{p(d)}$ = semivariance of portfolio; $\sigma_{ij(d)}$ = semicovariance between assets *i* and *j*; n = number of assets included in the portfolio

The model is formulated in matrix form to facilitate the calculations, as follows:

$$\sigma^2_{p(d)} = X^T \Omega(d) X \tag{8}$$

$$X^T = (\omega_1\ \omega_2\ \dots\ \omega_n) \tag{9}$$

where: $X = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix}$; $\Omega(d) = \begin{pmatrix} \sigma_{11(d)} & \sigma_{12(d)} & \cdots & \sigma_{1n(d)} \\ \sigma_{21(d)} & \sigma_{22(d)} & \cdots & \sigma_{2n(d)} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1(d)} & \sigma_{n2(d)} & \cdots & \sigma_{nn(d)} \end{pmatrix}$

$$\sigma_{p(d)} = \sqrt{X^T \Omega(d) X} \tag{10}$$

$$\sigma_{p(d)} = \sqrt{(\omega_1\ \omega_2\ \dots\ \omega_n) \cdot \begin{pmatrix} \sigma_{11(d)} & \sigma_{12(d)} & \cdots & \sigma_{1n(d)} \\ \sigma_{21(d)} & \sigma_{22(d)} & \cdots & \sigma_{2n(d)} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1(d)} & \sigma_{n2(d)} & \cdots & \sigma_{nn(d)} \end{pmatrix} \cdot \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix}} \tag{11}$$

The asset weights of the minimum-risk and target-return portfolios are determined using the Lagrange multiplier method. The portfolio optimization model is based on two key components: expected return and risk, with risk measured by semivariance.

For the minimum-risk portfolio, the optimization problem consists of minimizing portfolio risk subject to the budget constraint (Brătian, 2024; Brătian, 2018).

The associated Lagrangian function is defined as follows:

$$L = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i \omega_j \sigma_{ij(d)} + \lambda_1\left(\sum_{i=1}^{n} \omega_i - 1\right) \tag{12}$$

$$\begin{cases} \dfrac{\partial L}{\partial \omega_i} = 0 \\ \dfrac{\partial L}{\partial \lambda_1} = 0 \end{cases} \rightarrow \begin{cases} \displaystyle\sum_{j=1}^{n} \omega_i \, \sigma_{ij(d)} + \lambda_1 = 0 \\ \displaystyle\sum_{i=1}^{n} \omega_i = 1 \end{cases}$$

The resulting system of equations can be expressed in matrix form and solved to determine the optimal portfolio weights. The right-hand-side vector represents the first-order optimality conditions derived from the Lagrangian function together with the budget constraint (Brătian, 2024):

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \\ \lambda_1 \end{pmatrix} = \begin{pmatrix} \sigma_{11(d)} & \sigma_{12(d)} & \cdots & \sigma_{1n(d)} & 1 \\ \sigma_{21(d)} & \sigma_{22(d)} & \cdots & \sigma_{2n(d)} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{n1(d)} & \sigma_{n2(d)} & \cdots & \sigma_{nn(d)} & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \quad (13)$$

After determining the portfolio weights, the expected return of the minimum-risk portfolio is calculated as follows:

$$\mu_p = (\omega_1 \; \omega_2 \ldots \; \omega_n) \cdot \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} \quad (14)$$

where:

$$\mu_i = \tfrac{1}{T} \textstyle\sum_{t=1}^{T} r_{i,t} \quad (15)$$

The portfolio downside risk, measured by the downside deviation, is calculated as follows:

$$\sigma_{p(d)} = \sqrt{(\omega_1 \; \omega_2 \ldots \; \omega_n) \cdot \begin{pmatrix} \sigma_{11(d)} & \sigma_{12(d)} & \cdots & \sigma_{1n(d)} \\ \sigma_{21(d)} & \sigma_{22(d)} & \cdots & \sigma_{2n(d)} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1(d)} & \sigma_{n2(d)} & \cdots & \sigma_{nn(d)} \end{pmatrix} \cdot \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix}} \quad (16)$$

The portfolio optimization model is based on expected return and risk, measured by semivariance. For the target-return portfolio, the optimization problem consists of minimizing portfolio risk subject to the target expected return constraint and the budget constraint (Brătian, 2024).
The associated Lagrangian function is defined as follows:

$$L = \tfrac{1}{2} \textstyle\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_{ij(d)} + \lambda_1 \left(\sum_{i=1}^{n} \omega_i \, \mu_i - \mu_p \right) + \lambda_2 \left(\sum_{i=1}^{n} \omega_i - 1 \right) \quad (17)$$

$$\begin{cases} \dfrac{\partial L}{\partial \omega_i} = 0 \\ \dfrac{\partial L}{\partial \lambda_1} = 0 \\ \dfrac{\partial L}{\partial \lambda_2} = 0 \end{cases} \rightarrow \begin{cases} \displaystyle\sum_{j=1}^{n} \omega_j \, \sigma_{ij(d)} + \lambda_1 \mu_i + \lambda_2 = 0 \\ \displaystyle\sum_{i=1}^{n} \omega_i \, \mu_i = \mu_p \\ \displaystyle\sum_{i=1}^{n} \omega_i = 1 \end{cases}$$

The resulting system of equations can be expressed in matrix form and solved to obtain the optimal weights of the target-return portfolio (Brătian, 2024):

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \sigma_{11(d)} & \sigma_{12(d)} & \cdots & \sigma_{1n(d)} & \mu_1 & 1 \\ \sigma_{21(d)} & \sigma_{22(d)} & \cdots & \sigma_{2n(d)} & \mu_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \sigma_{n1(d)} & \sigma_{n2(d)} & \cdots & \sigma_{nn(d)} & \mu_n & 1 \\ \mu_1 & \mu_2 & \cdots & \mu_n & 0 & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \mu_p \\ 1 \end{pmatrix} \tag{18}$$

The portfolio downside risk, measured by the downside deviation, is calculated as follows:

$$\sigma_{p(d)} = \sqrt{(\omega_1\ \omega_2 \dots \omega_n) \cdot \begin{pmatrix} \sigma_{11(d)} & \sigma_{12(d)} & \cdots & \sigma_{1n(d)} \\ \sigma_{21(d)} & \sigma_{22(d)} & \cdots & \sigma_{2n(d)} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1(d)} & \sigma_{n2(d)} & \cdots & \sigma_{nn(d)} \end{pmatrix} \cdot \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix}} \tag{19}$$

In this analysis, portfolio performance is evaluated using the Sortino ratio, defined as the ratio of the portfolio return in excess of the Minimum Acceptable Return (MAR) to its downside deviation, as follows:

$$Sortino = \frac{\mu_p - MAR}{\sigma_{p(d)}} \tag{20}$$

Regarding the Minimum Acceptable Return (MAR), it was set equal to the risk-free rate, as this is generally considered an appropriate proxy for the risk-free investment alternative available to investors. The risk-free rate was approximated using the 12-month government securities fixing ask yield as of December 30, 2025 (5.93%, National Bank of Romania, 2025).

4. Results

In the following, we present the empirical research findings, which applies an extension of the Markowitz model within the mean-semivariance behavioral framework using companies listed on the Romanian capital market. The analysis covers the period from 18 December 2024 to 30 December 2025, and the empirical testing is conducted using the five companies selected from the constituents of the BET Index.
Before presenting the empirical analysis, the BET Index and the five selected companies are briefly introduced. The BET Index was chosen because it comprises the most liquid companies listed on the Romanian capital market, an essential characteristic for obtaining continuous and comparable data series required for portfolio optimization. The historical share price time series used in the analysis were obtained from the LSEG Workspace platform (LSEG, 2026).
The selected stocks – SNN (Nuclearelectrica), H2O (Hidroelectrica), BRD (BRD – Groupe Société Générale), FP (Fondul Proprietatea), and DIGI (Digi Communications N.V.) – were chosen because they represent key sectors of the economy, namely the energy, banking, and telecommunications sectors. These industries play a fundamental role in economic activity and provide an appropriate degree of sectoral diversification for portfolio construction.
All information presented below was obtained from the official website of the Bucharest Stock Exchange (BVB) and from the official websites of the companies included in the analysis.
The BET (Bucharest Exchange Trading) Index is the first index developed by the Bucharest Stock Exchange (BVB). It was launched on 19 September 1997 with a base value of 1,000 points and serves as the benchmark index of the Romanian capital market. The number of constituent companies is variable, ranging from a minimum of 10 to a maximum of 20. The BET Index was designed to provide a transparent and reliable benchmark for the regulated market operated by the BVB. Its performance can be replicated through a portfolio composed of the constituent stocks. The index methodology, including changes to its composition and constituent weights, is based on transparent qualitative rules established by the Bucharest Stock Exchange (Bursa de Valori București, 2026).

Societatea Națională Nuclearelectrica S.A. (ticker: SNN; Tax Identification Number: 10874881; Trade Register No. J1998007403409), headquartered at 48 Iancu de Hunedoara Boulevard, Sector 1, 011745 Bucharest, Romania, operates in the electric power generation industry (CAEN 3511). The company is the sole producer of nuclear energy in Romania and the country's only manufacturer of nuclear fuel for the CANDU 6 (Canada Deuterium Uranium) reactor technology. Established in 1998 and operating under the authority of the Romanian Ministry of Energy, Nuclearelectrica plays a strategic role in Romania's energy sector, making a significant contribution to the country's electricity generation and energy security (Nuclearelectrica, 2026).

S.P.E.E.H. Hidroelectrica S.A. (ticker: H2O; Tax Identification Number: 13267213; Trade Register No. J2000007426409), headquartered at 15–17 Ion Mihalache Boulevard, Floors 10–15, Sector 1, Bucharest, Romania, operates in the renewable electricity generation industry (CAEN 3512). The company's principal activity is the generation of electricity from renewable energy sources. Established in 2000, Hidroelectrica is one of Romania's largest electricity producers and occupies a strategic position within the national energy sector. Its operations are primarily focused on the generation of hydroelectric power, which has positioned the company as one of the most significant issuers listed on the Bucharest Stock Exchange (BVB) (Hidroelectrica, 2026).

BRD – Groupe Société Générale S.A. (ticker: BRD; Tax Identification Number: 361579; Trade Register No. J1991000608402), headquartered at BRD Tower, 1–7 Ion Mihalache Boulevard, Sector 1, Bucharest, Romania, operates in the other monetary intermediation sector (CAEN 6419). Established in 1991, BRD is one of the leading banking institutions in Romania. It is the second-largest bank in the country by total assets and has the fourth-largest market capitalization on the Bucharest Stock Exchange (BVB). As a member of the Société Générale Group, BRD holds a prominent position within the Romanian banking sector, providing a broad range of financial services to both retail and corporate clients (BRD – Groupe Societe Generale, 2026).

Fondul Proprietatea S.A. (ticker: FP; Tax Identification Number: 18253260; Trade Register No. J2005021901408), headquartered at 76–80 Buzești Street, 7th Floor, Sector 1, Bucharest, Romania, operates in the mutual funds and similar financial entities sector (CAEN 6430). Established in 2005, Fondul Proprietatea is one of the largest investment funds listed on the Bucharest Stock Exchange (BVB). The Fund's primary activity consists of managing equity holdings in strategically important Romanian companies, particularly those operating in the energy and infrastructure sectors (Fondul Proprietatea, 2026).

Digi Communications N.V. (ticker: DIGI; Tax Identification Number: 5888716; Trade Register No. J1994012278401), is incorporated in Amsterdam, the Netherlands, with its Romanian headquarters located at 75 Dr. Nicolae Staicovici Street, Forum 2000 Building, 4th Floor, Sector 5, Bucharest, Romania. The company operates in the wired telecommunications activities sector (CAEN 6110). Established in 1994, Digi Communications is one of the leading providers of telecommunications services in Romania and the broader region. Its operations encompass cable television, fixed broadband internet, mobile telephony, and electronic communications services. The company occupies a prominent position within the telecommunications sector and is widely regarded as a growth-oriented company, characterized by continuous investment in infrastructure and the ongoing expansion of its service portfolio (DIGI, 2026).

Furthermore, I applied an extension of the Markowitz portfolio optimization model based on the mean–semivariance framework, in which I identified the minimum-risk portfolio, defined as the portfolio with the lowest semivariance.

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \\ \lambda_1 \end{pmatrix} = \begin{pmatrix} 0.000086 & 0.000034 & 0.000033 & 0.000059 & 0.000022 & 1 \\ 0.000034 & 0.000077 & 0.000030 & 0.000029 & 0.000023 & 1 \\ 0.000033 & 0.000030 & 0.000056 & 0.000028 & 0.000023 & 1 \\ 0.000059 & 0.000029 & 0.000028 & 0.000904 & 0.000023 & 1 \\ 0.000022 & 0.000023 & 0.000023 & 0.000023 & 0.000041 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\\ \omega_5\\ \lambda_1\end{pmatrix}=\begin{pmatrix}16795.64 & -4278.60 & -7221.13 & -726 & -4569.91 & 0.09\\ -4278.60 & 17824.81 & -6423.19 & -47.47 & -7075.54 & 0.13\\ -7221.13 & -6423.19 & 27373.81 & -78.77 & -13650.72 & 0.23\\ -726 & -47.47 & -78.77 & 1159.74 & -307.50 & 0.00\\ -4569.91 & -7075.54 & -13650.72 & -307.50 & 25603.68 & 0.54\\ 0.09 & 0.13 & 0.23 & 0.00 & 0.54 & 0.00\end{pmatrix}\cdot\begin{pmatrix}0\\0\\0\\0\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\\ \omega_5\\ \lambda_1\end{pmatrix}=\begin{pmatrix}9.45\%\\ 12.78\%\\ 23.42\%\\ 0.47\%\\ 53.88\%\\ -3.26377\text{E}-05\end{pmatrix}$$

Based on the results obtained, the composition of the minimum-risk portfolio is as follows: SNN = 9.45%; BRD = 12.78%; DIGI = 23.42%; FP = 0.47%; H2O = 53.88%.
For this portfolio structure, the expected return and the minimum risk are:

$$\mu_{\text{RiskMIN}}=(9.45\%\ \ 12.78\%\ \ 23.42\%\ \ 0.47\%\ \ 53.88\%)\cdot\begin{pmatrix}0.11\%\\0.15\%\\0.21\%\\0.21\%\\0.01\%\end{pmatrix}=0.08501\%$$

The expected daily return obtained above is annualized, resulting in an annual expected return of approximately 21.42%, to facilitate meaningful comparisons.

$$\sigma^2_{RiskMIN(d)}=(9.45\%\ \ 12.78\%\ \ 23.42\%\ \ 0.47\%\ \ 53.88\%)\cdot$$
$$\cdot\begin{pmatrix}0.000086 & 0.000035 & 0.000033 & 0.000059 & 0.000022\\ 0.000035 & 0.000077 & 0.000030 & 0.000029 & 0.000023\\ 0.000033 & 0.000030 & 0.000056 & 0.000028 & 0.000023\\ 0.000059 & 0.000029 & 0.000028 & 0.000905 & 0.000023\\ 0.000022 & 0.000023 & 0.000023 & 0.000023 & 0.000041\end{pmatrix}\cdot\begin{pmatrix}9.45\%\\12.78\%\\23.42\%\\0.47\%\\53.88\%\end{pmatrix}$$
$$=0.0032638\%$$

$$\sigma_{RiskMIN(d)}=\sqrt{\sigma^2_{RiskMIN(d)}}=0.57129\%$$

The portfolio downside deviation is 0.57129% on a daily basis, reflecting the dispersion of returns below the Minimum Acceptable Return (MAR). This corresponds to an annualized downside deviation of approximately 9.05%.
For the performance analysis, the Sortino ratio is calculated as follows:

$$Sortino=\frac{0.2142-0.0593}{0.0905}=1.71$$

A Sortino ratio of approximately 1.71 indicates that the portfolio generates 1.71 units of excess return for each unit of downside risk assumed, suggesting favorable risk-adjusted performance from the perspective of downside risk.

The next step was to identify the efficient portfolio (*EP*) corresponding to a target annual return of 30%, taking the minimum-risk portfolio as the reference point.

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\\ \omega_5\\ \lambda_1\\ \lambda_2\end{pmatrix}=\begin{pmatrix}0.000086 & 0.000034 & 0.000033 & 0.000059 & 0.000022 & 0.11\% & 1\\ 0.000034 & 0.000077 & 0.000030 & 0.000029 & 0.000023 & 0.15\% & 1\\ 0.000033 & 0.000030 & 0.000056 & 0.000028 & 0.000023 & 0.21\% & 1\\ 0.000059 & 0.000029 & 0.000028 & 0.000904 & 0.000023 & 0.21\% & 1\\ 0.000022 & 0.000023 & 0.000023 & 0.000023 & 0.000041 & 0.01\% & 1\\ 0.11\% & 0.15\% & 0.21\% & 0.21\% & 0.01\% & 0 & 0\\ 1 & 1 & 1 & 1 & 1 & 0 & 0\end{pmatrix}^{-1}\cdot\begin{pmatrix}0\\ 0\\ 0\\ 0\\ 0\\ 0.12\%\\ 1\end{pmatrix}$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\\ \omega_5\\ \lambda_1\\ \lambda_2\end{pmatrix}=\begin{pmatrix}16519.58 & -3882.16 & -5013.01 & -648.87 & -6975.65 & -56.64 & 0.14\\ -3882.16 & 17255.48 & -9594.26 & -158.33 & -3620.67 & 81.34 & 0.06\\ -5013.01 & -9594.26 & 9711.58 & -696.42 & 5592.23 & 453.06 & -0.15\\ -648.76 & -158.39 & -696.53 & 1138.03 & 365.55 & 15.85 & -0.01\\ -6975.65 & -3620.67 & 5592.23 & 365.59 & 4638.54 & -493.61 & 0.96\\ -56.64 & 81.34 & 453.06 & 15.86 & -493.61 & -11.62 & 0.01\\ 0.14 & 0.06 & -0.15 & -0.01 & 0.96 & 0.01 & 0.00\end{pmatrix}\cdot\begin{pmatrix}0\\ 0\\ 0\\ 0\\ 0\\ 0.12\%\\ 1\end{pmatrix}$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\\ \omega_5\\ \lambda_1\\ \lambda_2\end{pmatrix}=\begin{pmatrix}7.49\%\\ 15.59\%\\ 39.08\%\\ 1.02\%\\ 36.82\%\\ -0.004015569\\ -2.9224\mathrm{E}-05\end{pmatrix}$$

Based on the optimization results, the composition of the efficient portfolio corresponding to the target annual return of 30% is as follows: SNN = 7.49%, BRD = 15.59%, DIGI = 39.08%, FP = 1.02%, and H2O = 36.82%.

$$\sigma^2_{EP(d)} = (7.49\%\ \ 15.59\%\ \ 39.08\%\ \ 1.02\%\ \ 36.82\%)\cdot$$
$$\cdot\begin{pmatrix}0.000086 & 0.000035 & 0.000033 & 0.000059 & 0.000022\\ 0.000035 & 0.000077 & 0.000030 & 0.000029 & 0.000023\\ 0.000033 & 0.000030 & 0.000056 & 0.000028 & 0.000023\\ 0.000059 & 0.000029 & 0.000028 & 0.000905 & 0.000023\\ 0.000022 & 0.000023 & 0.000023 & 0.000023 & 0.000041\end{pmatrix}\cdot\begin{pmatrix}7.49\%\\ 15.59\%\\ 39.08\%\\ 1.02\%\\ 36.2\%\end{pmatrix}$$
$$= 0.0034025\%$$

$$\sigma_{EP(d)} = \sqrt{\sigma^2_{EP(d)}} = 0.58331\%$$

The portfolio downside deviation is 0.58331% on a daily basis, reflecting the dispersion of returns below the minimum acceptable return (MAR). This corresponds to an annualized downside deviation of approximately 9.24%.

For the performance analysis, the Sortino ratio is calculated as follows:

$$Sortino = \frac{0.30 - 0.0593}{0.0924} = 2.60$$

A Sortino ratio of approximately 2.60 indicates that the portfolio generates 2.60 units of excess return for each unit of downside risk, suggesting favorable downside risk-adjusted performance.

5. Discussion and Conclusion

The results confirm the principles of Post-Modern Portfolio Theory, according to which portfolio optimization is achieved by balancing expected return with downside risk. The composition of the analyzed portfolios illustrates how asset allocation changes depending on the investment objective, namely minimizing risk or achieving a specified target return.

For the minimum-risk portfolio, which exhibits a downside risk of 9.05%, Hidroelectrica (H2O) accounts for the largest allocation (53.88%), reflecting its relatively low downside risk during the sample period. This finding highlights its role as the portfolio's primary stabilizing asset. The remaining assets Nuclearelectrica (SNN) (9.45%), BRD (12.78%), and Digi Communications (DIGI) (23.42%) contribute to portfolio diversification with smaller allocations, while Fondul Proprietatea (FP) receives only 0.47%, reflecting its comparatively higher level of downside risk.

This portfolio generates an annual expected return of approximately 21.42%. It should also be noted that, from the 20 constituent companies of the BET Index, only those exhibiting relatively low pairwise return correlations, illustrated in Figure 1, and high market liquidity were selected in order to maximize the benefits of diversification. However, this figure represents an ex-ante expected return, estimated from historical arithmetic means, whereas the BET Index's approximately 45% appreciation over the same period reflects realized, ex-post performance of all 20 constituent companies. Since the portfolio analyzed here comprises only 5 of these 20 companies, selected specifically for diversification and downside-risk minimization purposes, its performance is not directly comparable to that of the full index differences in composition and risk profile alone would account for a substantial part of the gap. Within this context, the expected return of approximately 21.42% can be regarded as reasonable given the portfolio's defensive investment objective.

BRD	DIGI	FP	H2O	
0.39176	**0.21532**	**0.14905**	**0.19462**	SNN
	0.36567	**0.06790**	**0.30476**	BRD
		0.12528	**0.29792**	DIGI
			0.05593	FP

Figure 1. The correlations among the log returns of the assets included in the portfolio.
Source: Authors' analysis

For the efficient portfolio constructed to achieve a target annual return of 30%, the asset allocation changes substantially. The weight of Hidroelectrica (H2O) decreases to 36.82%, while the allocation to Digi Communications (DIGI) increases markedly to 39.08%. This reallocation reflects a shift toward assets with higher expected return potential, which is accompanied by a higher level of investment risk. The evolution of DIGI's share price during the sample period from approximately RON 64.08 to RON 110.20 supports this allocation, highlighting its ability to generate superior returns.

In addition, BRD increases its portfolio weight slightly to 15.59%, suggesting a balanced risk–return profile, whereas Nuclearelectrica (SNN) experiences a moderate reduction to 7.49%. Fondul Proprietatea (FP) remains only marginally represented, with a weight of 1.02%, confirming that its relatively high downside risk negatively affects its inclusion in optimal portfolios.

This portfolio is associated with a downside risk of approximately 9.24%, calculated relative to a minimum acceptable return (MAR) of 5.93%.

The graphical representations of returns above and below the minimum acceptable return for the selected financial assets are presented in the Appendices (I, II, III, IV, V).

Consistent with the existing literature, the findings support the view that portfolio diversification contributes to reducing overall investment risk and that portfolio composition is strongly influenced by the individual characteristics of the constituent assets. The results clearly illustrate the role of defensive assets, such as Hidroelectrica (H2O), in mitigating portfolio risk, and the contribution of growth-oriented assets, such as Digi Communications (DIGI), to enhancing expected returns.

From a behavioral perspective, the change in portfolio composition indicates that investors are willing to accept a higher level of downside risk in exchange for the prospect of achieving higher returns.

In conclusion, the analysis demonstrates that, as the target return increases, the optimal portfolio shifts toward riskier assets with greater return potential. These findings are consistent with the principles of PMPT and support its applicability to the Romanian capital market.

That being said, Figure 2 below illustrates the price evolution of the stocks analyzed in this paper (the evolution of daily closing prices over the analyzed period).

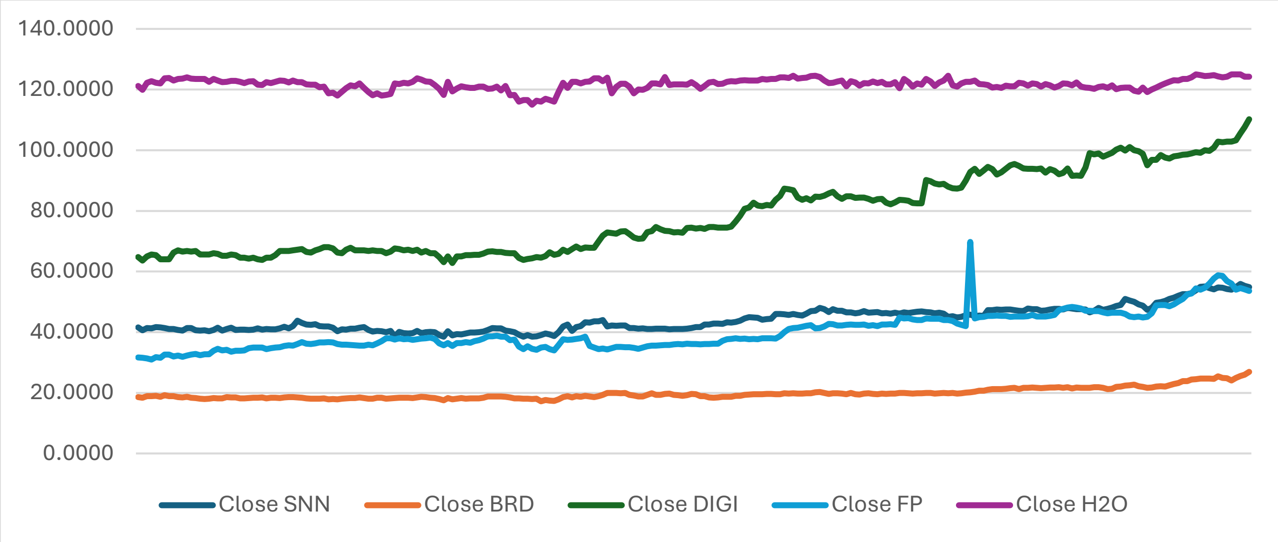


Figure 2. The evolution of stock prices during the period 18 December 2024 – 30 December 2025.
Source: Author's analysis

The first part of this paper presents the theoretical framework. A substantial section is devoted to MPT, developed by Harry Markowitz, which constitutes the conceptual foundation of PMPT, later advanced by Frank A. Sortino. This section also introduces the concepts of the efficient frontier and the Sortino ratio as a measure of risk-adjusted performance. The second part describes the models employed to determine the optimal portfolio structure based on the risk–return trade-off. The final section presents the empirical case study, which represents the main contribution of this research: the application of Post-Modern Portfolio Theory to the Romanian capital market.

The results obtained from applying the mean–semivariance extension of the Markowitz model confirm the applicability of Post-Modern Portfolio Theory. The composition of the two portfolios, the minimum-risk portfolio and the target-return portfolio adjusts according to the investment objectives and constraints imposed during the optimization process.

The empirical analysis was conducted using five companies selected from the twenty constituents of the BET Index: Nuclearelectrica, representing the energy sector; BRD – Groupe Société Générale, representing the banking sector; Digi Communications N.V., a major telecommunications company; Fondul Proprietatea, one of the largest closed-end investment funds listed on the Bucharest Stock Exchange; and Hidroelectrica, an energy company specializing in hydroelectric power generation. The analysis was carried out within the framework of Post-Modern Portfolio Theory, using the risk-free rate as the minimum acceptable return (MAR). Based on the downside deviation, the semivariance and semicovariance of the selected financial assets were calculated in order to construct the semivariance–semicovariance matrix. Subsequently, the Lagrange multiplier method was applied to determine the optimal portfolio weights, together with the associated levels of downside risk and expected return. In addition, portfolio performance was evaluated using the Sortino ratio.

In conclusion, the findings demonstrate that portfolio composition is directly influenced by the risk–return trade-off and confirm the applicability of Post-Modern Portfolio Theory to the Romanian capital market. Investors should pay particular attention to diversification, not only across economic sectors but also across different asset classes, including equities, bonds, precious metals, bank deposits, and real estate investments, among others.

Future research could extend this analysis by comparing the standard Markowitz mean–variance model with its mean–semivariance extension to determine whether the use of semivariance produces sufficiently superior results to justify the additional computational complexity. Another potential direction would be to extend the analysis by considering a larger sample of financial assets and incorporating transaction costs into the optimization framework, thereby providing a more realistic assessment of the model's practical applicability.

5.1. Limitations of study

This study is subject to several limitations. First, the analysis relies on a relatively short sample period of 252 daily observations (18 December 2024 – 30 December 2025), which may not be sufficient to capture the full range of market conditions or to produce stable estimates of semivariance and semicovariance; results obtained during this specific bull-market period may not generalize to other market phases. Second, the portfolio was constructed using only 5 of the 20 constituent companies of the BET Index, which, while adequate for illustrating the mean-semivariance methodology, restricts the diversification potential and the representativeness of the results relative to the broader market. Third, the analysis does not include a direct comparison with a traditional mean-variance (Markowitz) portfolio constructed from the same asset sample, which would have provided a clearer benchmark for assessing the practical benefits of the mean-semivariance approach. Finally, the model does not account for transaction costs, taxes, or liquidity constraints, which could affect the feasibility and net performance of the proposed portfolios in practice.

Funding: This research received no external funding.
Conflicts of Interest: The authors state that they have no conflicts of interest.

References

Brătian, V. R., 2018. Portfolio Optimization. Application of the Markowitz Model Using Lagrange and Profitability Forecast. *Expert Journal of Economics*, 6(1), pp. 26-34.

Brătian, V. R., Bucur, A. & Opreana, C., 2016. *Finanțe cantitative- Evaluarea valorilor mobiliare și gestiunea portofoliului*. Sibiu: Editura Universitații "Lucian Blaga" din Sibiu.

Brătian, V., 2024. Portfolio optimization under the Mean-Semivariance Behavioral Hypothesis. Empirical evidence of the dependence between optimal portfolio structure and ESG Risk Scores. *Management of Sustainable Development*, 16(2), pp. 13.

BRD - Groupe Societe Generale, 2026. BRD - Groupe Societe Generale. [Online]. Available at: https://www.brd.ro/ [Accessed March 2026].

Bursa de Valori București, 2026. Bursa de Valori București. [Online]. Available at: https://bvb.ro/. [Accessed 20 January 2026].

Cheng, P. & Wolverton, M. L., 2001. MPT and the Downside Risk Framework. *The Journal of Real Estate Portfolio Management*, 7(2), p. 125-132.

DIGI, 2026. digi-communications. [Online]. Available at: https://www.digi-communications.ro/. [Accessed March 2026].

Estrada, J., 2008. Mean-Semivariance Optimization: A Heuristic Approach. *Journal of Applied Finance,* 18(1), pp. 57–72.

Fondul Proprietatea, 2026. Fondul Proprietatea. [Online]. Available at: https://www.fondulproprietatea.ro/home.html/. [Accessed March 2026].

Harville, D., 2023. Modern and post-modern portfolio theory as applied to moneyline betting. *Journal of Quantitative Analysis in Sports*, 19(2), pp. 73-89.

Hidroelectrica, 2026. Hidroelectrica. [Online]. Available at: https://www.hidroelectrica.ro/. [Accessed March 2026].

Jianu, I., Geambasu, C., Sova, R. & Geambasu, L., 2013. Risk measurement in post-modern portfolio theory: Differences from modern portfolio theory. *Economic Computation and Economic Cybernetics Studies and Research*, 47(1), pp. 113-132.

Kahneman, D. & Tversky, A., 1979. Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, 47(2), pp. 263-292.

Kahraman, S.R. & Somuncu, K., 2023. Portfolio optimization with semi-variance model: An application on BIST-100 Index. *The Journal Business Science*, 11(3), pp. 214-226.

LSEG, 2026. LSEG Workspace. London Stock Exchange Group. [Online]. Available at: https://www.lseg.com/en. [Accessed March 2026].

Mangram, M., 2013. A Simplified Perspective of the Markowitz Portfolio Theory. Global Journal of *Business Research*, 7(1), pp. 59-70.

Markowitz, H., 1959. *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons.

National Bank of Romania, 2025. *Government Securities Market Data / Interactive Database*. [Online]. Available at: www.bnr.ro [Accessed March 2026].

Nuclearelectrica, 2026. Nuclearelectrica. [Online]. Available at: https://www.nuclearelectrica.ro/. [Accessed March 2026].

Şerban, R. A., Mihaiu, D. M., Brătian, V., Opreana, A., & Herciu, M, 2025. Effectiveness of the ESG approach in portfolio selection – an empirical evidence from the US stock market. *Journal of Business Economics and Management*, 26(4), pp. 918-940

Sortino, F. & Price, L., 1994. Performance Measurement in a Downside Risk Framework. *The Journal of Investing Fall*, 3(3), pp. 59-64.

Sortino, F. & Van der Meer, R., 1991. Downside risk: Capturing what's at stake in investment situations. *Journal of Portfolio Management*, 17(4), pp. 27-31.

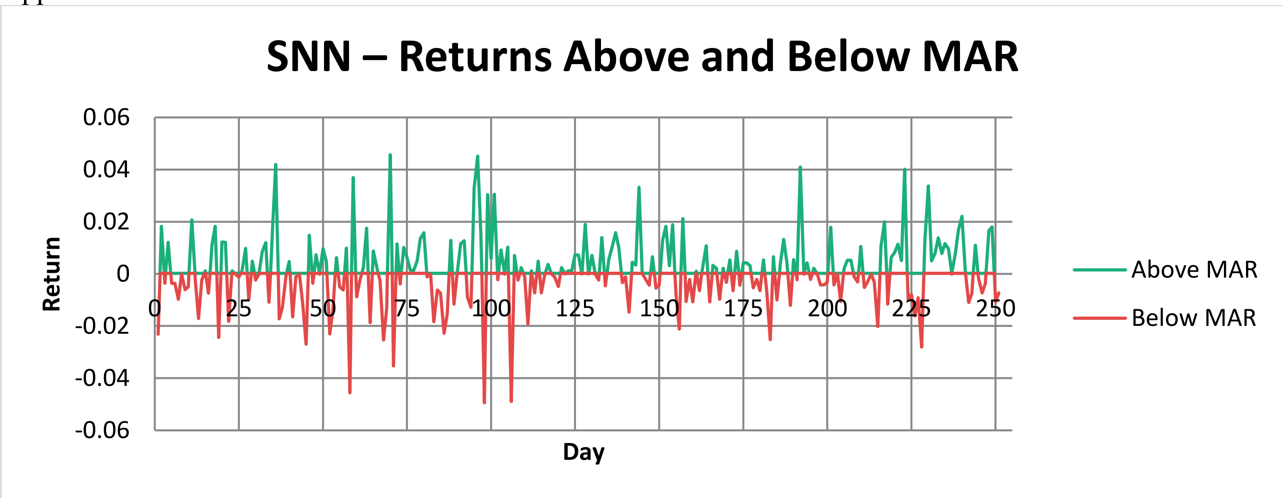
Stancu, I. & Stancu, D., 2012. *Finanțe corporative cu Excel*. Bucureşti: Editura Economică.

Tsai, M. F. & Wang, C. J., 2012. Post-Modern Portfolio Theory for Information Retrieval. *Procedia Computer Science*, 13(1), pp. 80-85.

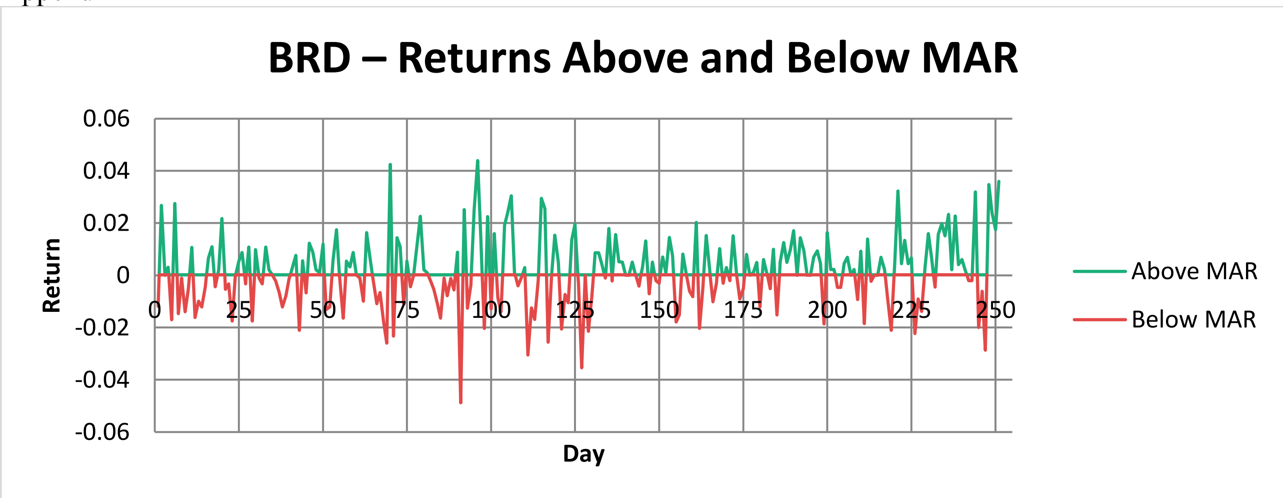
Venkataraman, S. V., 2023. A remark on mean-semivariance behaviour: Downside risk and capital asset pricing. *International Journal of Finance & Economics*, 28(3), pp. 13.

Appendices

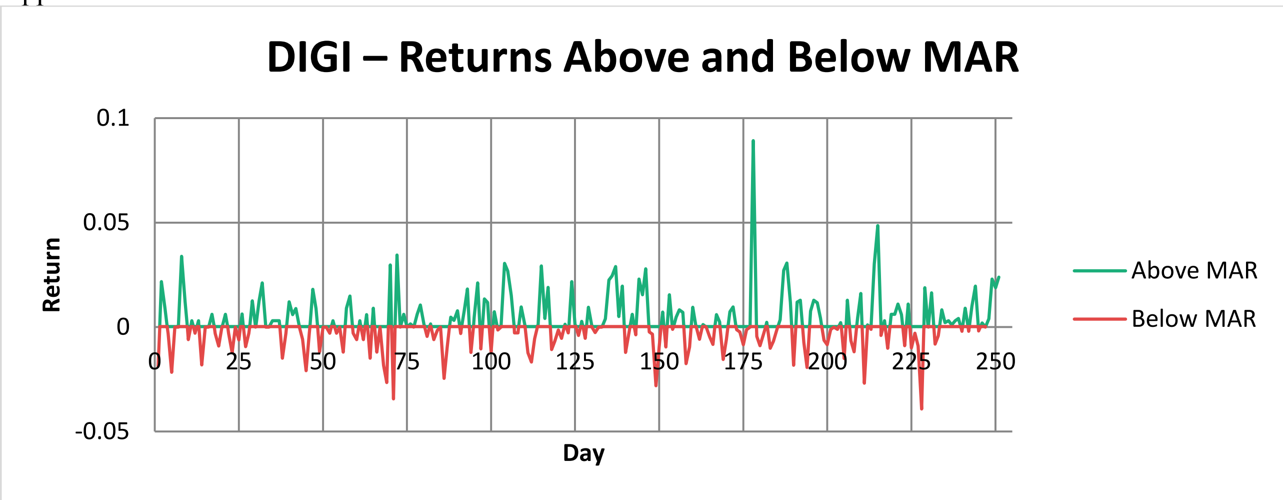
Appendix I



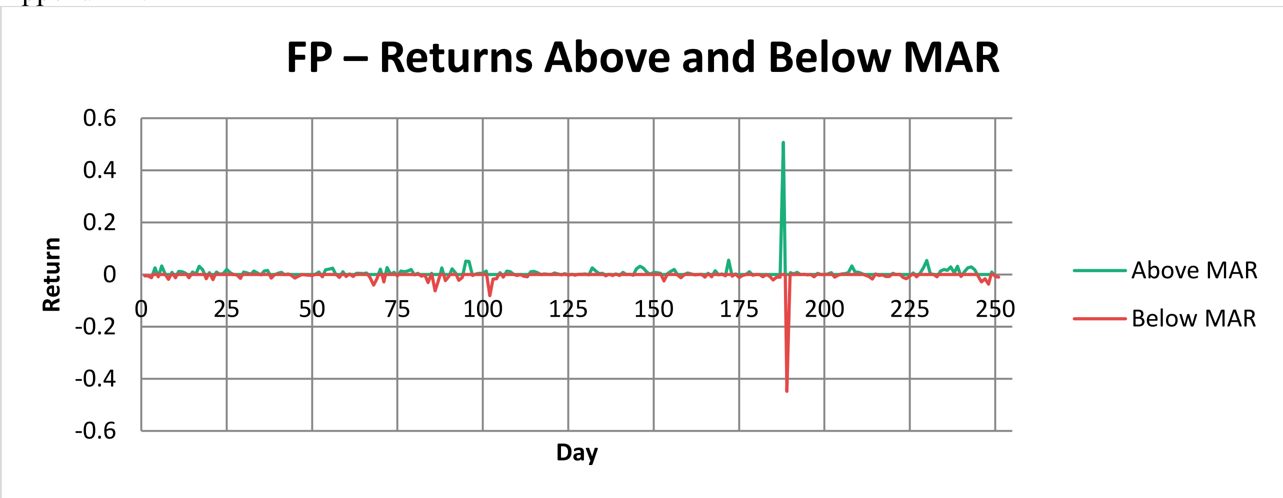
Appendix II



Appendix III



Appendix IV



Appendix V