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Human Capital, Wealth, and Renewable Resources

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This paper studies dynamic interdependence among physical capital, resource and human capital. We integrate the Solow one-sector growth, Uzawa-Lucas two-sector and some neoclassical growth models with renewable resource models. The economic system consists of the households, production sector, resource sector and education sector. We take account of three ways of improving human capital: Arrow's learning by producing (Arrow, 1962), Uzawa's learning by education (Uzawa, 1965), and Zhang's learning by consuming (Zhang, 2007). The model describes a dynamic interdependence among wealth accumulation, human capital accumulation, resource change, and division of labor under perfect competition. We simulate the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also examine effects of changes in the productivity of the resource sector, the utilization efficiency of human capital, the propensity to receive education, and the propensity to save upon dynamic paths of the system.

Keywords*: education; physical capital; renewable resource; human capital; propensities to save and to learn; time distribution among study, work and leisure*

JEL Classification*: O41; I25; Q2;*

1. Introduction

Three kinds of "capital" - physical capital such as machines, human capital such as skills, and renewable resources such as forests - are important for economic growth and development. As human capital, resources and physical capital are scarce resources and play different roles in production and consumption, it is significant to study how these resources are allocated in different activities. Moreover, these stock variables change according to different mechanisms. Physical capital changes due to, for instance, depreciation and wealth accumulation. Savings by households, firms, or nations are essential for physical capital accumulation. Human capital is accumulated through human capital in learning. Education and learning by doing are common sources of human capital accumulation. Stock of renewable resources is also changeable according how fast agents utilize resources and how fast renewable resources grow. This paper studies dynamic interdependence among physical capital, resource and human capital. We integrate the Solow one-sector growth, Uzawa-Lucas two-sector and some neoclassical growth models with renewable resource models. The economic system consists of the households, production sector, resource sector and education sector. We take account of three ways of improving human capital: Arrow's learning by producing (1962), Uzawa's learning by education (Uzawa, 1965), and Zhang's learning by consuming (2007). The model describes a dynamic interdependence among wealth accumulation, human capital accumulation, resource change, and division of labor under perfect competition.

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As far as physical capital and wealth accumulation are concerned, the model in this study is based on the neoclassical growth theory. Most of the models in the neoclassical growth theory are extensions and generalizations of the pioneering works of Solow in 1956. The model has played an important role in the development of economic growth theory by using the neoclassical production function and neoclassical production theory. The Solow model has been extended and generalized in numerous directions (e.g., Uzawa, 1961; Kurz, 1963; Diamond, 1965; Stiglitz, 1967; Drugeon and Venditti, 2001; Erceg *et al*. 2005). An important direction of extending the traditional neoclassical one-sector growth model was carried out by Uzawa (1965), who proposed a formal dynamic growth model with education. But with regards to formal modeling of education and economic growth, the work by Lucas (1988) has recently caused a great interest in the issue among economists. Dynamic interdependence between education and economic growth is currently a main topic in the literature of economic theory and economic empirical studies (e.g., Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; Fleisher *et al.* 2011; Li *et al.*, 2012; Castelló-Climent and Hidalgo-Cabrillana, 2012). In the Uzawa-Lucas model and many of their extensions and generalizations, it is implicitly assumes that all skills and human capital is formed due to formal schooling. Common sense tells us that much of the so-called human capital may be accumulated through parents' influences, family and other social environment, and other social and economic activities, not to say learning by producing (and professional training). If these non-school factors are neglected in modelling human capital and economic growth, we may not be able to properly understand the role of formal education in economic development. Chen and Chevalier (2008) point out: "Making and exploiting an investment in human capital requires individuals to sacrifice not only consumption, but also leisure. When estimating the returns to education, existing studies typically weigh the monetary costs of schooling (tuition and forgone wages) against increased wages, neglecting the associated labor/leisure tradeoff." This study will generalize the Uzawa-Lucas two-sector growth model by taking account of leisure activities, learning by producing and learning by consuming.

Natural resources are incorporated into the neoclassical growth theory in the 1970s (e.g., Plourde, 1970, 1971; Stiglitz, 1974; Clark, 1976; Dasgupta and Heal, 1979). In fact, economists were aware of the necessity of modeling resources with dynamic theory long before. For instance, Gordon (1956) emphasized the need for a dynamic approach to fisheries economics as one finds in capital theory in economics: "The conservation problem is essentially one which requires a dynamic formulation… The economic justification of conservation is the same as that of any capital investment – by postponing utilization we hope to increase the quantity available for use at a future date. In the fishing industry we may allow our fish to grow and to reproduce so that the stock at a future date will be greater than it would be if we attempted to catch as much as possible at the present time. … [I]t is necessary to arrive at an optimum which is a catch per unit of time, and one must reach this objective through consideration of the interaction between the rate of catch, the dynamics of fish population, and the economic time-preference schedule of the community or the interest rate on invested capital. His is a very complicated problem and I suspect that we will have to look to the mathematical economists for assistance in clarifying it." As pointed out by Munro and Scott (1985), in the 1950s it was quite difficult to develop workable dynamic models of resources. Solow (1999) also argues for the necessity of taking account of natural resources in the neoclassical growth theory. According to Solow if the resource good is used as one of the inputs in the production, then it is easy to incorporate the use of renewable resources into the neoclassical growth model. Nevertheless, Solow does not show how to incorporate possible consumption of renewable resource into the growth model. There are only a few models of growth and renewable resources which treat the renewable resource as both input of production and a source of utility (see, Beltratti, *et al*., 1994, Ayong Le Kama, 2001). Our model contains the renewable resource as a source of utility and input of production. It should be noted that there are also studies on dynamic interactions among economic growth, renewable resources and elastic labor supply on the basis of the neoclassical growth theory with capital accumulation and renewable resource (e.g., Eliasson and Turnovsky, 2004, Alvarez-Cuadrado and van Long, 2011). Our model differs from these studies not only in that we use an alternative utility function, but also in that we introduce human capital and education sector into the growth theory with capital and resource.

Another important variable in dynamic analysis is time distribution among various activities. The allocation of time has been explicit introduced into economic theory since Becker (1965) published his seminal work in 1965. There is an immense body of empirical and theoretical literature on economic growth with time distribution between home and non-home economic and leisure activities (e.g., Benhabib and Perli, 1994; Ladrón-de-Guevara *et al*. 1997; Jones and Manuelli, 1995; Turnovsky, 1999; Greenwood and Hercowitz, 1991; Rupert *et al*. 1995; Cambell and Ludvigson, 2001). Nevertheless, only a few theoretical economic growth models with renewable resource and human capital explicitly treat work time as an endogenous variable. This paper introduces endogenous time into the neoclassical growth theory with renewable resource. This paper is to integrate two papers by Zhang (2007, 2011). The former paper deals with education and capital accumulation, while the latter studies dynamic interactions between resource and physical capital. This paper integrates the two models to examine dynamic interactions among human capital, physical capital and renewable resources. Our model is also a synthesis of three main growth models – Solow's one-sector growth model, Arrow's learning by doing model, and the Uzawa-Lucas's growth model with education - in the growth literature. We integrate the main mechanisms of economic growth in these three models in a comprehensive framework. The remainder of the paper is organized as follows. Section 2 defines the economic model with endogenous human capital accumulation, resource dynamics and wealth accumulation. Section 3 shows that the motion of the economic system is described by three differential equations and simulates the model. Section 4 carries out comparative dynamics analysis. Section 5 concludes the study.

2. The Basic Model

The economy has three - production, education and renewable resource - sectors. Most aspects of the production sector are similar to the standard one-sector growth model in the neoclassical growth theory (Burmeister and Dobell, 1970; Barro and Sala-i-Martin, 1995). It is assumed that there is only one (durable) good in the economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use inputs such as labor with varied levels of human capital, different kinds of capital, knowledge and natural resources to produce material goods or services. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. We assume a homogenous and fixed population $\bar{N}$. The labor force is employed the three sectors. We select commodity to serve as numeraire, with all the other prices being measured relative to its price. We assume that wage rate is identical among all professions.

2.1. The production sector

We assume that production is to combine labor force, $N_i(t)$, and physical capital, $K_i(t)$, and renewable resource, $X_i(t)$. We use the conventional production function to describe a relationship between inputs and output. Let $F_i(t)$ stand for output level of the production sector at time t. The production function is specified as follows

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t) X_i^{\gamma_i}(t), \quad A_i,\ \alpha_i,\ \beta_i,\ \gamma_i > 0, \quad \alpha_i + \beta_i + \gamma_i = 1, \tag{1}$$

where A_i, α_i, β_i and γ_i are positive parameters. Markets are competitive; thus labor and capital earn their marginal products. The rate of interest, $r(t)$, and wage rate, $w(t)$, the price of the resource, $p_x(t)$, are determined by markets. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \quad p_x(t) = \frac{\gamma_i F_i(t)}{X_i(t)}, \tag{2}$$

where δ_k is the given depreciation rate of physical capital.

2.2. Resource sector and change of renewable resources

We use $X(t)$ to represent the stock of the resource. We assume that the natural growth rate of the resource is a logistic function of the existing stock (e.g., Brander and Taylor, 1998; Brown, 2000; Hannesson, 2000; Cairns and Tian, 2010, Farmer and Bednar-Friedl, 2011). It should be noted that there are some alternative approaches to renewable resources in the literature (Tornell and Velasco, 1992; Long and Wang, 2009; Fujiwara, 2011). The logistic function is

$$\phi_0 X(t)\left(1 - \frac{X(t)}{\phi}\right),$$

where the variable, ϕ, is the maximum possible size for the resource stock, called the carrying capacity of the resource, and , the variable, ϕ_0, is "uncongested" or "intrinsic" growth rate of the renewable resource. If the stock is equal to ϕ, then the growth rate should equal zero. If the carrying capacity is much larger than the current stock, then the growth rate per unit of the stock is approximately equal to the intrinsic growth rate. In this case, the congestion effect is negligible. It should be noted that according Jinni (2006), the carrying capacity changes as a function of the stock of a renewable resource. Also in Benchekroun (2003), an inversed-V shaped dynamics of resource accumulation is accepted. The resource decreases if its stock is sufficiently large. There are also models which introduce human efforts and other factors to the dynamics of resources (e.g., Long, 1977; Berck, 1981; Levhari and Withagen, 1992; Ayong Le Kama, 2001; Wirl, 2004).

We use $F_x(t)$ to stand for the harvest rate of the resource. The change rate in the stock is then equal to the natural growth rate minus the harvest rate, that is

$$\dot{X}(t) = \phi_0 X(t)\left(1 - \frac{X(t)}{\phi}\right) - F_x(t). \tag{3}$$

We assume a nationally owned open-access renewable resource. The open-access case was initially examined by Gordon (1954). There are different approaches to growth with renewable resources with different property-rights regimes (e.g., Bulter and Barbier, 2005; Copeland and Taylor, 2009; Alvarez-Guadrado and Von Long, 2011; Tajibaeva, 2012). With open access, harvesting occurs up to the point at which the current return to a representative entrant equals the entrant's cost. We use $N_x(t)$ and $K_x(t)$ to respectively stand for the labor force and capital stocks employed by the resource sector. We assume that harvesting of the resource is carried out according to the following harvesting production function

$$F_x(t) = A_x X^b(t) K_x^{\alpha_x}(t) N_x^{\beta_x}(t), \quad A_x, b \geq 0, \quad \alpha_x, \beta_x > 0, \quad \alpha_x + \beta_x = 1, \tag{4}$$

where A_x, b, α_x and β_x are parameters. It should be noted that the Schaefer harvesting production function which is taken on the following form

$$F_x(t) = A_x X(t) N_x(t),$$

is a special case of (4). The Schaefer production function does not take account of capital (or with capital being fixed, see Schaefer, 1957). The function with fixed capital and technology is widely applied to fishing (see also, Paterson and Wilen, 1977; Milner-Gulland and Leader-Williams, 1992; Bulter and van Kooten, 1999). As machines are important inputs in harvesting, we explicitly take account of capital input.

Harvesting is carried out by competitive profit-maximizing firms under conditions of free entry. The marginal conditions are given as follows

$$r(t) + \delta_k = \frac{\alpha_x p_x(t) F_x(t)}{K_x(t)}, \quad w(t) = \frac{\beta_x p_x(t) F_x(t)}{N_x(t)}. \tag{5}$$

2.3. The education sector and accumulation of human capital

We assume that the education sector is also characterized of perfect competition. Students are supposed to pay the education fee $p_e(t)$ per unity time. The education sector pays teachers and capital with the market rates. Let $N_e(t)$ and $K_e(t)$ stand for respectively the labor force and capital stocks employed by the education sector. The cost of the education sector is given by $w(t)N_e(t) + r(t)K_e(t)$. The total education service is measured by the total (qualified) education time received by the population. The production function of the education sector is assumed to be a function of $K_e(t)$ and $N_e(t)$. We specify the production function of the education sector as follows

$$F_e(t) = A_e K_e^{\alpha_e}(t) N_e^{\beta_e}(t), \ \alpha_e, \beta_e > 0, \ \alpha_e + \beta_e = 1, \tag{6}$$

where A_e, α_e and β_e are positive parameters. Empirical studies on education production functions are referred to, for instance, Krueger (1999). For given $p_e(t)$, $H(t)$, $r(t)$, and $w(t)$, the education sector chooses $K_e(t)$ and $N_e(t)$ to maximize profit. The optimal solution is given by

$$r(t) + \delta_k = \frac{\alpha_e \, p_e(t) F_e(t)}{K_e(t)}, \ \ w(t) = \frac{\beta_e \, p_e(t) F_e(t)}{N_e(t)}. \tag{7}$$

Following Zhang (2007), we assume that there are three sources of improving human capital, through education, "learning by producing", and "learning by leisure". Arrow (1962) first introduced learning by doing into growth theory; Uzawa (1965) took account of trade-offs between investment in education and capital accumulation, and Zhang (2007) introduced impact of consumption on human capital accumulation (via the so-called creative leisure) into growth theory. We use $H(t)$ to stand for the level of human capital. We propose that human capital dynamics is given by

$$\dot{H}(t) = \frac{\upsilon_e F_e^{a_e}(t)\left(H^m(t) T_e(t) \bar{N}\right)^{b_e}}{H^{\pi_e}(t)\bar{N}} + \frac{\upsilon_i F_i^{a_i}(t)}{H^{\pi_i}(t)\bar{N}} + \frac{\upsilon_x F_x^{a_x}(t)}{H^{\pi_x}(t)\bar{N}} + \frac{\upsilon_h C^{a_h}(t) T_h^{b_h}(t)}{H^{\pi_h}(t)\bar{N}} - \delta_h H(t), \tag{8}$$

where $\delta_h \ (> 0)$ is the depreciation rate of human capital, υ_e, υ_i, υ_h, a_e, b_e, a_i, a_h and b_h are non-negative parameters. The signs of the parameters π_e, π_i, and π_h are not specified as they can be either negative or positive. The above equation is a synthesis and generalization of Arrow's, Uzawa's, and Zhang's ideas about human capital accumulation. The term, $\upsilon_e F_e^{a_e}\left(H^m T_e \bar{N}\right)^{b_e} / H^{\pi_e} \bar{N}$, describes the contribution to human capital improvement through education. Human capital tends to increase with an increase in the level of education service, F_e, and in the (qualified) total study time, $H^m T_e \bar{N}$. The population $\bar{N}$ in the denominator measures the contribution in terms of per capita. The term H^{π_e} indicates that as the level of human capital of the population increases, it may be more difficult (in the case of $\pi_e > 0$) or easier, for instance, due to learning externalities as in Choi (2011) (in the case of $\pi_e < 0$) to accumulate more human capital via formal education. We refer the literature on human capital externalities to Rauch (1993) and Liu (2007), and on economies of scale and scope in education to Cohn and Cooper (2004). It should be noted that this unique formation of human capital is important to explore complexity of human capital accumulation, division of time and economic growth. For instance, the formation implies that if a society can enable people to learning through work experiences and through non-higher-education activities, national economic growth can be sustainable if its higher education is not efficient.

We take account of learning by doing effects in human capital accumulation by the term $\upsilon_i F_i^{a_i} / H^{\pi_i}$. This term implies that contribution of the production sector to human capital improvement is positively related to its production scale F_i and is dependent on the level of human capital. The term H^{π_i} takes account of returns to scale effects in human capital accumulation. The case of $\pi_i > (<) \ 0$ implies that as human capital is increased it is more difficult (easier) to further improve the level of human capital. We take account of learning by consuming by the term $\upsilon_c C^{a_h} T_h^{b_h} / H^{\pi_h} N_0$. This term can be interpreted similarly as the term for learning by producing. It should be noted that in the literature on education and economic growth, it is assumed that human capital evolves according to the following equation (see Barro and Sala-i-Martin, 1995)

$$\dot{H}(t) = H^{\eta}(t) G(T_e(t)),$$

where the function G is increasing as the effort rises with $G(0) = 0$. In the case of $\eta < 1$, there is diminishing return to the human capital accumulation. This formation is due to Lucas (1988). As $\dot{H}/H < H^{\eta-1} G(1)$, we conclude that the growth rate of human capital must eventually tend to zero no matter how much effort is devoted to accumulating human capital. Uzawa's model may be considered a special case of the Lucas model with $\gamma = 0$, $U(c) = c$, and the assumption that the right-hand side of the above equation is linear in the effort. Solow adapts the Uzawa formation to the following form

$$\dot{H}(t) = H(t)\kappa T_e(t).$$

This is a special case of the above equation. The new formation implies that if no effort is devoted to human capital accumulation, then $\dot{H}(0) = 0$ (human capital does not vary as time passes; this results from depreciation of human capital being ignored); if all effort is devoted to human capital accumulation, then $g_H(t) = \kappa$ (human capital grows at its maximum rate; this results from the assumption of potentially unlimited growth of human capital). Between the two extremes, there is no diminishing return to the stock $H(t)$. To achieve a given percentage increase in $H(t)$ requires the same effort. As remarked by Solow (2000), the above formulation is very far from a plausible relationship. If we consider the above equation as a production for new human capital (i.e., $\dot{H}(t)$), and if the inputs are already accumulated human capital and study time, then this production function is homogenous of degree two. It has strong increasing returns to scale and constant returns to $H(t)$ itself. It can be seen that our approach is more general to the traditional formation with regard to education. Moreover, we treat teaching also as a significant factor in human capital accumulation. Efforts in teaching are neglected in Uzawa-Lucas model. Choi (2011) proposes the following human capital accumulation equation

$$\dot{H}(t) = B(t)[u(t)H(t)]^{\phi}\,\overline{H}^{\theta}(t) - \delta_H\, H(t),$$

where $B(t)$ is productivity of human capital production and $u(t)$ is the fraction of human capital devoted to human capital accumulation. Here $\overline{H}(t)$, is the average human capital stock in the economy. The term, $\overline{H}^{\theta}(t)$, measures learning externalities. As for a homogenous population, $\overline{H}(t)$ is $H(t)$. We see that Choi's learning equation is a special case of (3).

2.4. Consumer behaviors

Consumers make decisions on choice of consumption levels of goods, services, and education (which is services), as well as on how much to save. It should also be remarked that neither Uzawa nor Lucas took account of leisure in their growth models with education. Hahn (1990) takes account of leisure in generalizing the Lucas model, altering model to the case that each member of the population can use his available – nevertheless fixed - time for working, for leisure, or for studying. Like Hahn, this study also introduces leisure into the growth model with leisure, but in an alternative approach to household proposed by Zhang (1993). We denote per capita wealth by $\bar{k}(t)$, where $\bar{k}(t) \equiv K(t)/\overline{N}$. Per capita current income from the interest payment $r(t)\bar{k}(t)$ and the wage payment $w(t)T(t)$ is given by

$$y(t) = r(t)\bar{k}(t) + w(t)H^m(t)T(t).$$

We call $y(t)$ the current income in the sense that it comes from consumers' work and current earnings from ownership of wealth. The total value of wealth that consumers can sell to purchase goods and to save is equal to $p_0(t)\bar{k}(t)$, where $p_0(t)\ (=1)$ is the price of the capital good (which is unity). Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by

$$\hat{y}(t) = y(t) + \bar{k}(t) = (1 + r(t))\bar{k}(t) + w(t)H^m(t)T(t). \tag{9}$$

The disposable income is used for saving and consumption. At each point of time, a consumer would distribute the total available budget among saving, $s(t)$, consumption of the commodity, $c(t)$, education, $T_e(t)$, and consumption of the resource good, $c_x(t)$. The budget constraint is given by

$$c(t) + s(t) + p_e(t)T_e(t) + p_x(t)c_x(t) = \hat{y}(t) = (1 + r(t))\bar{k}(t) + w(t)H^m(t)T(t). \tag{10}$$

The total available time is allocated among working, receiving education, and leisure. The consumer is faced with the following time constraint

$$T(t) + T_e(t) + T_h(t) = T_0, \tag{11}$$

where T_0 is the total available time. Substituting (10) into the budget constraint (7) yields

$$\begin{aligned} &c(t) + s(t) + w(t)H^m(t)T_h(t) + \bar{p}(t)T_e(t) + p_x(t)c_x(t) = \bar{y}(t) = (1 + r(t))\bar{k}(t) + w(t)H^m(t), \\ &\bar{p}(t) \equiv p_e(t) + w(t)H^m(t). \end{aligned} \tag{12}$$

At each point of time, consumers have four variables, the consumption level of consumption good $c(t)$, the consumption level of resource $c_x(t)$, the level of saving $s(t)$, the leisure time $T_h(t)$, and the education time $T_e(t)$, to decide. For simplicity of analysis, we specify the utility function as follows

$$U(t) = T_h^{\sigma_0}(t)\, T_e^{\eta_0}(t) c^{\xi_0}(t) s^{\lambda_0}(t) c_x^{\chi_0}(t), \quad \sigma_0, \eta_0, \xi_0, \lambda_0, \chi_0 > 0, \tag{13}$$

where σ_0 is called the propensity to use leisure time, ξ_0 the propensity to consume the good, λ_0 the propensity to own wealth, σ_0 the propensity to use leisure time, and η_0 the propensity to get education, and χ_0 the propensity to consume the resource good. It should be noted that we enter the time that the household spends on education into the utility. In traditional economic growth theory with endogenous human capital, education is mainly modeled by assuming that it positively affects earnings through enhanced productivity. Nevertheless, common sense tells us that one chooses education not only for higher wages, but also for social status, for social network buildings, signaling, or other purposes. In the literature of education and economics, the signaling view of education was initially formally presented by Spence (1973), Arrow (1973), and Stiglitz (1975). This implies that in addition to wages there are many other factors which we should take account of when analyzing decision on education decision. For instance, Lee (2007) holds that signaling explains why American students study more in college than in high school while the opposite is true for East Asian students. Hussey (2012) empirically distinguish human capital augmentation and the signaling value of MBA education using U.S. data. Hussey shows that signaling plays a large role in producing post-graduation earnings. Applying the idea that money burning (such as some advertising activities by firms, e.g., Nelson, 1974; Kihlstrom and Riordian, 1984; Milgrom and Roberts, 1986) may convey credible information, with a model of higher education as money burning activities Ishida (2004) shows: "this money burning activity can actually be welfare-improving under certain conditions. This result indicates that, even when education is simply a way to waste resources, it can still be meaningful and even socially desirable under certain conditions."

For the representative consumer, the wage rate $w(t)$, the rate of interest $r(t)$, the fee of education $p_e(t)$, and the price of resource $p_x(t)$ are given in markets. Maximizing $U(t)$ subject to the budget constraint yields

$$w(t)\, \mathrm{H}^m(t)T_h(t) = \sigma\, \bar{y}(t), \quad \bar{p}(t)T_e(t) = \eta\, \bar{y}(t), \quad c(t) = \xi\, \bar{y}(t), \quad s(t) = \lambda\, \bar{y}(t), \quad p_x(t)c_x(t) = \chi\, \bar{y}(t), \tag{14}$$

where $\sigma \equiv \rho\sigma_0$, $\eta \equiv \rho\eta_0$, $\xi \equiv \rho\xi_0$, $\lambda \equiv \rho\lambda_0$, $\chi \equiv \rho\chi_0$, $\rho \equiv \dfrac{1}{\sigma_0 + \eta_0 + \xi_0 + \lambda_0 + \chi_0}$.

The demand for resource is given by $c_x = \chi \, \bar{y} / p_x$. The demand decreases in its price and increases in the disposable income. An increase in the propensity to consume the resource good increases the consumption when the other conditions are fixed. As any factor is related to all the other factors over time, it is difficult to see how one factor affects any other variables over time in the dynamic system.

We now find dynamics of capital accumulation. According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t) = \lambda \, \bar{y}(t) - \bar{k}(t). \tag{15}$$

For the education sector, the demand and supply balances at any point of time

$$T_e(t)\bar{N} = F_e(t). \tag{16}$$

"The research indicates that literacy scores, as a direct measure of human capital, perform better in growth regressions than indicators of schooling. A country able to attain literacy scores 1% higher than the international average will achieve levels of labour productivity and GDP per capita that are 2.5 and 1.5% higher, respectively, than those of other countries." (OECD, Education at a Glance, 2006: 155). This implies that when modeling education and economic growth, it is necessary to take quantity and quality aspects of education. Equation (16) accounts for quantity balance of education. The quality aspect of education is reflected in the term of human capital accumulation associated with education in equation (3).

2.5. Full employment of the production factors

The labor force and capital are allocated among the three sectors. Let $N(t)$ and $K(t)$ stand for respectively the labor supply and total capital stock. The total labor force and the total capital are given by

$$N(t) = H^m(t)T(t)\bar{N}, \quad K(t) = \bar{N}\,\bar{k}(t), \tag{17}$$

where the parameter, m, measures of the efficiency that the population applies human capital. The conditions of full employment of labor and capital are

$$K_i(t) + K_e(t) + K_x(t) = K(t), \quad N_i(t) + N_e(t) + N_x(t) = N(t). \tag{18}$$

As output of the production sector is equal to the sum of the level of consumption, the depreciation of capital stock and the net savings, we have

$$C(t) + S(t) - K(t) + \delta_k \, K(t) = F_i(t), \tag{19}$$

where $C(t)$ is the total consumption, $S(t) - K(t) + \delta_k \, K(t)$ is the sum of saving and depreciation, and

$$C(t) = c(t)\bar{N}, \quad S(t) = s(t)\bar{N}.$$

As the resource output is used up by the production sector and the households, we have

$$c_x(t)\bar{N} + X_i(t) = F_x(t). \tag{20}$$

We completed the model. The model is based on some strict assumptions. Nevertheless, from the structural point of view our model is general in the sense that it synthesizes a few well-known models in economics. For instance, if we neglect resource and assume human capital constant, then the model is the one-sector neoclassical growth model by Solow (1956). If we neglect resources, then the model is structurally similar to the well-known Uzawa-Lucas two-sector model (Uzawa, 1965; Lucas, 1988). As mentioned before, our approach is also based on some growth models in the literature of resource economics.

3. The Dynamics and Its Properties

The dynamic system consists of three differential equations for wealth (or physical capital), human capital and resource stock. As the three differential equations contain other variables, we need to find three differential equations which contain only three variables. The following lemma shows how to obtain the three differential equations which contain only three variables. We also provide a computational procedure for calculating all the variables in the system at any point of time. This section examines dynamics of the model. The following lemma provides the procedure about how to determine the motion of all the variables in the dynamic system. We first introduce a variable

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}.$$

3.1. Lemma

The dynamics of the economic system is governed by the following three differential equations with three variables, $z(t)$, $X(t)$, and $H(t)$

$$\begin{aligned} \dot{z}(t) &= \Lambda_z\left(z(t),\ X(t),\ H(t)\right), \\ \dot{X}(t) &= \Lambda_X\left(z(t),\ X(t),\ H(t)\right), \\ \dot{H}(t) &= \Lambda_H\left(z(t),\ X(t),\ H(t)\right), \end{aligned} \tag{21}$$

where Λ_z, Λ_X and Λ_H are $z(t)$, $X(t)$, and $H(t)$ given in the appendix. Moreover, all the other variables are determined as functions of $z(t)$, $X(t)$, and $H(t)$ at any point of time by the following procedure: $x_i(t)$ by (A6) $\rightarrow p_x(t)$ by (A5) $\rightarrow r(t)$ by (A3) $\rightarrow w(t)$ by (A3) $\rightarrow \bar{k}(t)$ by (A20) $\rightarrow K(t) = \bar{k}(t)\bar{N}$ $\rightarrow N(t)$ by (A18) $\rightarrow T(t) = N(t)/H^m(t)\bar{N}$ $\rightarrow T_h(t)$ and $T_e(t)$ by (A16) $\rightarrow$ $p_w(t)$ by the definition $\rightarrow$ $p_e(t)$ by (A16) $\rightarrow K_i(t)$ and $K_e(t)$ by (A13) $\rightarrow K_x(t)$ by (A11) $\rightarrow N_i(t)$, $N_e(t)$, and $N_x(t)$ by (A1) $\rightarrow \bar{y}(t)$ by (A15) $\rightarrow$ $c_x(t)$, $c(t)$, $s(t)$ by (14) $\rightarrow X_i(t) = x_i(t)N_i(t) \rightarrow F_i(t)$ by (1) $\rightarrow F_x(t)$ by (4) $\rightarrow F_e(t)$ by (6) $\rightarrow U(t)$ by (11).

The lemma provides a computational procedure for following the motion of the economic system with initial conditions. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model. In the remainder of this study, we specify the depreciation rates by $\delta_k = 0.05$, $\delta_h = 0.03$, and let $T_0 = 1$. We specify the other parameters as follows

$$\begin{aligned} &\alpha_i = 0.33,\ \gamma_i = 0.08,\ \alpha_e = 0.45,\ \alpha_x = 0.3,\ \phi_0 = 5,\ \phi = 8,\ \lambda_0 = 0.6,\ \xi_0 = 0.08, \\ &\sigma_0 = 0.2,\ \eta_0 = 0.01,\ \chi_0 = 0.01,\ N_0 = 5,\ A_i = 1,\ A_e = 0.9,\ A_x = 0.3,\ b = 0.5, \\ &m = 0.8,\ v_e = 1,\ v_i = 2,\ v_h = 1.2,\ v_x = 1.5,\ a_e = 0.3,\ b_e = 0.4,\ a_i = 0.4, \\ &a_x = 0.1,\ a_h = 0.2,\ b_h = 0.1,\ \pi_e = 0.3,\ \pi_i = 0.7,\ \pi_x = 0.7,\ \pi_h = 0.6. \end{aligned} \tag{22}$$

The propensity to save is 0.6 and the propensities to consume education and resource are 0.01. We specify the values of the parameters, α_i and α_x in the Cobb-Douglas productions approximately 0.3. The propensity to enjoy leisure is 0.2. The total productivities of the production sector, education sector, and resource sector are respectively 1, 0.9 and 0.3. The conditions $\pi_e = 0.2$, $\pi_i = 0.7$ and $\pi_h = 0.1$ mean respectively that the learning by education, learning by producing, and learning by consuming exhibits (weak) increasing effects in human capital. We plot the motion of the system under (22) with the following initial conditions

$$z(0) = 0.08, \quad X(0) = 7, \quad H(0) = 12.$$

The motion of the variables is plotted in Figure 1. In Figure 1, the national output is

$$Y = F_i + p_x F_x + p_e F_e.$$

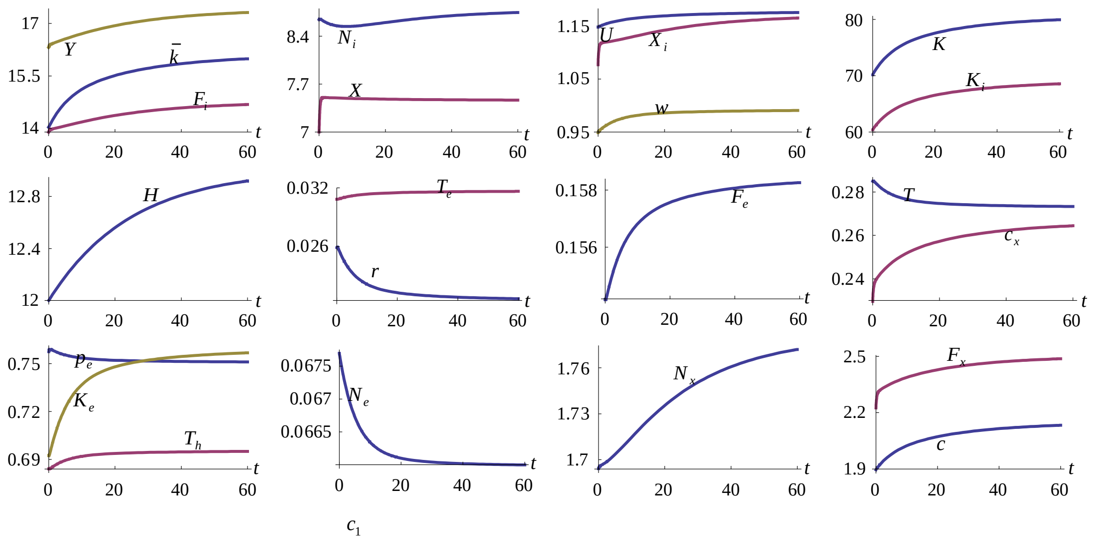

Figure 1. The Motion of the Economic System

As the initial level of human capital is lower than its equilibrium value, human capital rises over time. In association with rises in human capital, the wage rate rises and rate of interest falls over time. The equilibrium values of the variables are listed as follows

$$Y = 17.33, \quad K = 80.16, \quad H = 12.93, \quad N = 10.53, \quad X = 7.47, \quad F_i = 14.70,$$
$$F_x = 2.49, \quad F_e = 0.16, \quad N_i = 8.74, \quad N_x = 1.77, \quad N_e = 0.066, \quad K_i = 68.72,$$
$$K_x = 10.68, \quad K_e = 0.76, \quad r = 0.021, \quad p_e = 0.75, \quad p_x = 1.01, \quad W = 7.69, \quad T = 0.27,$$
$$T_h = 0.70, \quad T_e = 0.032, \quad c = 2.14, \quad c_x = 0.27, \quad s = 16.03, \quad U = 1.18.$$

It is straightforward to calculate the three eigenvalues at the equilibrium point as follows

-4.50, -0.18, -0.04.

As the eigenvalues are negative, the equilibrium point is locally stable. Hence, if the system is near the equilibrium, it will approach the equilibrium in the long term. This conclusion is important as it guarantees that we can effectively carry out comparative dynamic analysis.

4. Comparative dynamic analysis

We simulated the motion of the national economy under (22). We now study how the economic system reacts to exogenous changes, for instance, in resource capacity and preference. As the lemma gives a computational procedure to calibrate the motion of all the variables, we can conduct analysis on effects of change in any parameter on transitory processes as well stationary states of all the variables. In the rest of this study we use $\overline{\Delta} x(t)$ to stand for the change rate of the variable, $x(t)$, in percentage due to changes in the parameter value.

4.1. A rise in the carrying capacity of the renewable resource

We first study the case when the carrying capacity of renewable resource is increased as follows: $\phi: 8 \Rightarrow 8.2$. The simulation result is plotted in Figure 2. When he capacity is expanded, from equation (3) we see that the level of renewable resource stock tends to increase. The old development path is disturbed. The level of resource stock is augmented. In association of rises in the stock, both the production sector and households use more resources. The price of resource stock is lowered due to the expansion of supply. The education time and leisure time are initially reduced and work time is initially reduced; in the long term the time distribution is slightly affected. It should be noted that education time is augmented in the long term. The level of human capital is initially increased faster than the total labor force; in the long term the level of human capital is increased less than the total labor force. The total physical stock is also increased in association with rises in the consumption level of goods and the wealth. The education sector's output and its inputs are slightly affected. The other two sectors' output levels and inputs are increased. The utility level and national output are enhanced.

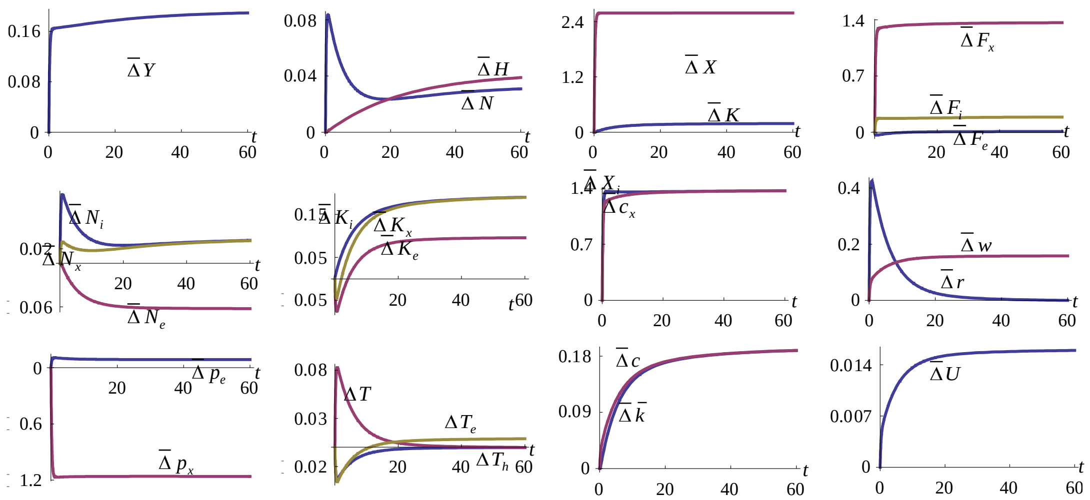

Figure 2. A Rise in the Capacity of Resources

Our simulation shows that if the economic system functions effectively, an economy with richer natural resources should have faster economic growth and better steady state. It should be mentioned that the impact of natural resources on economic as well as human development has caused attention of economists for a long time. Debates about whether natural resources are a blessing or a curse for human development are still a hot topic in the literature of economic development. It is well-known that the in the 1990s Sachs and Warner (1999, 2001) demonstrated a negative relationship between resource dependence and economic growth over the period 1970-1990. Since then, the curse of natural resource hypothesis has been theoretically re-examined and empirically tested in many studies. In a recent comprehensive study on natural resources and economic development, Daniele (2011: 568) concludes: "Natural resources can be a blessing for countries, but the blessing can turn into a curse when rents serve to fund conflicts, to corrupt institutions or are simply wasted. So, the effects that resources produce on people's welfare do not appear to depend on the resources themselves, as much as on the social and institutional ability to manage them. In this respect, the concept of resource curse appears misleading, as it tends to hide the real pathology affecting some nations: poor governance of natural resources." In fact, It has been empirically demonstrated that natural resources may have either an adverse or positive effect on the equilibrium growth rate (for instance, Gylfason, *et al*. 1999, Barbier, 1999, Chen and Lu, 2009).

4.2. An enhancement in efficiency of the education sector

We now increase the total productivity of the education sector as follows: $A_e: 0.8 \Rightarrow 0.92$. When A_e is increased, by $T_e N_0 = F_e$ the education time is increased initially. In the association of rise in the productivity, the price of education tends to fall and the education sector employs less labor and capital inputs. The households spend more time on education and less time on work and leisure. The three sectors' output levels are all increased. The national output and wealth initially fall slightly and increases but very small in the long term. The total labor is reduced and the level of human capital and the wage rate are increased. The rate of interest falls and the utility level is enhanced.

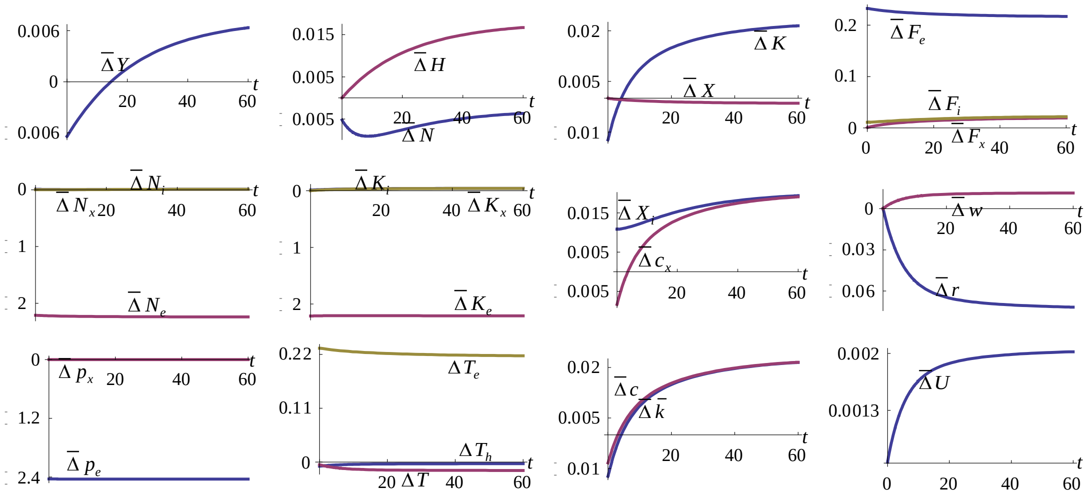

Figure 3. An Enhancement in Efficiency of the Education Sector

4.3. Human capital being more effectively utilized

We now study what will happen to the economic system if workers more effectively utilize human capital as follows: $m: 0.8 \Rightarrow 0.82$. The total labor is increased. The increase in the total labor is mostly absorbed by the production and resource sectors. The output levels and capital inputs of the production and resource sectors are increased. The output level and two inputs of the education sector are slightly affected. The economy has lower level of the resource, even though the resource input and consumption levels are increased. The rate of interest is initially increased and reduced in the long term. The wage rate is reduced. The education price is initially slightly increased, but reduced in the long term. The price of the resource is increased. The households spend more time on education and less time on leisure. The work time is slightly affected. The national output and utility level are enhanced.

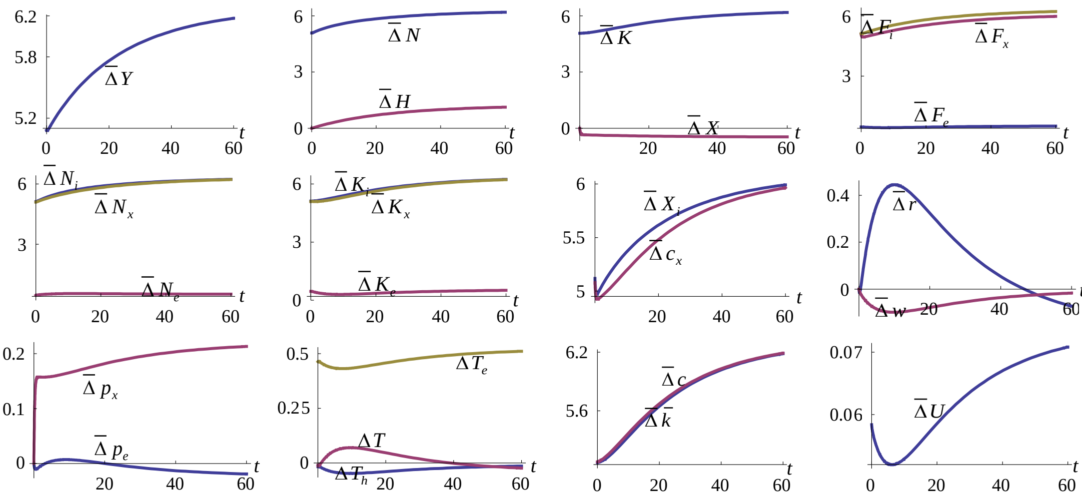

Figure 4. Human Capital Being More Effectively Utilized

4.4. The propensity to receive education being strengthened

We increase the propensity to receive education as follows: $\eta_0 : 0.01 \Rightarrow 0.012$. The simulation result is plotted in Figure 5. As the preference for education is strengthened, the education time is increased. Both leisure time and work time are reduced. The level of human capital is increased. The total labor supply is initially reduced and increased in the long term. The fall in the total labor is due to the reduction the work time. Correspondingly, the national output falls initially and rises in the long term. The rise in the demand for education drives up the price of education. The output level and capital and labor inputs of the education sector are increased. The total wealth is slightly changed. The stock of renewable resource is reduced first and then increased. The price of resources is reduced. The consumption and input levels of the resources are initially reduced and enhanced in the long term. The output levels and input levels of the production and resource sectors are slightly reduced. The rate of interest and utility level are increased.

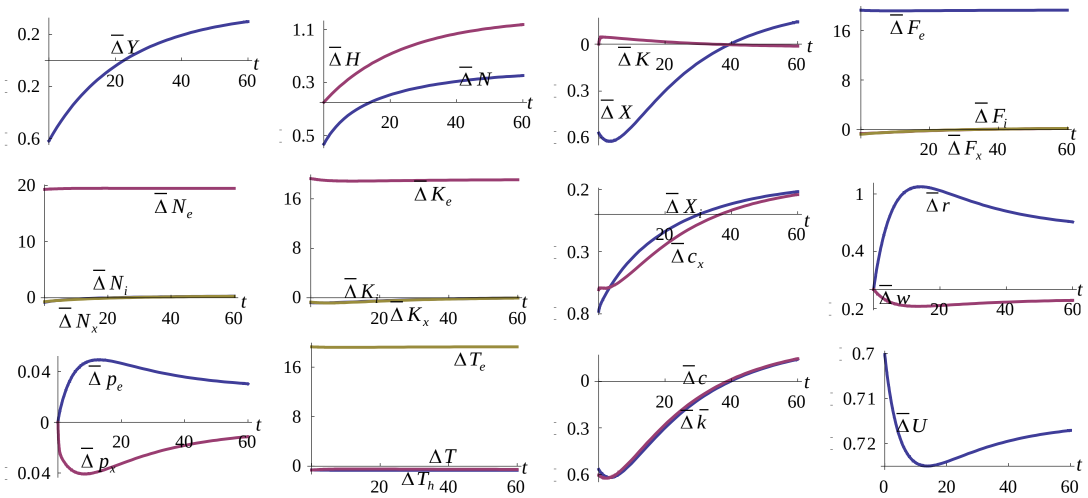

Figure 5. The Propensity to Receive Education Being Strengthened

4.5. The propensity to save being augmented

We increase the propensity to save as follows: $\lambda_0 : 0.6 \Rightarrow 0.62$. The simulation result is plotted in Figure 6. As the propensity to save is increased, the national wealth is increased. The increase in the total wealth enables the three sectors employs more capital inputs in the long term. The rise in the total physical wealth is association with a slight fall in the stock of the renewable resource. The price of the resource is increased, while the price of education is reduced. The rate of interest is reduced, while the wage rate is increased. The households work longer hours and have less leisure time and education time. The consumption level of resource by the households is initially reduced, and increased in the long term. The output levels of the production and resource sectors are increased, while the output level of the education sector is slightly reduced in association with falling in the price of education.

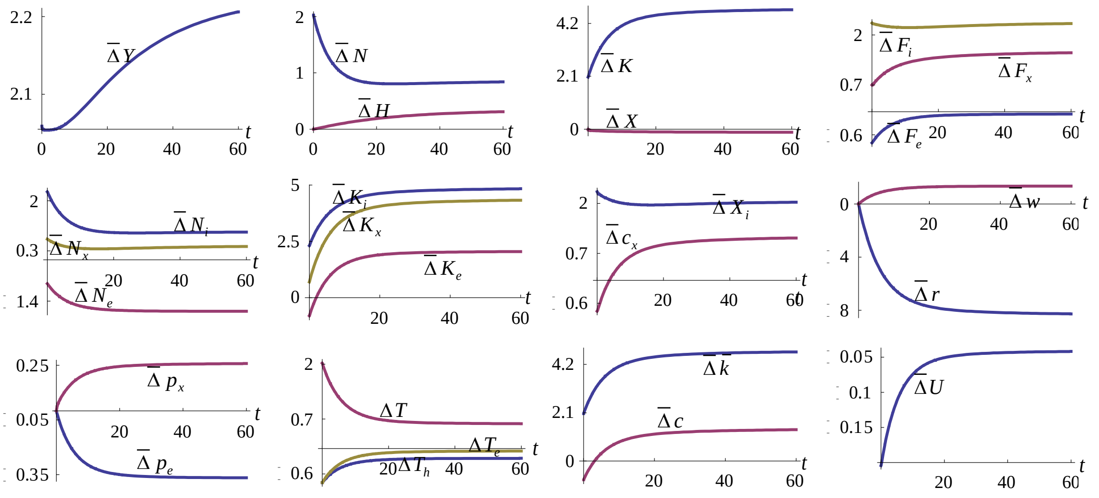

Figure 6. The Propensity to Save Being Augmented

5. Concluding Remarks

The main concern of this paper is dynamic interdependence among physical capital, resource and human capital. We modelled the dynamics of the three variables in an economic system with production, resource and education sectors. We took account of three ways of improving human capital: learning by producing, learning by education, and learning by consuming. The model describes a dynamic interdependence among wealth accumulation, human capital accumulation, resource change, and division of labor under perfect competition. We simulated the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also examined effects of changes in the productivity of the resource sector, the utilization efficiency of human capital, the propensity to receive education, and the propensity to save upon dynamic paths of the system. We may extend the model in some directions. For instance, we may introduce some kind of government intervention in education into the model. Ownership of resources is a complicated issue. Another interesting extension is to examine how human capital and education may interact with population dynamics.

6. References

Arrow, K.J., 1962. The Economic Implications of Learning by Doing. *Review of Economic Studies*, 29, pp.155-173.

Arrow, K., 1973. *Higher Education as a Filter. Journal of Public Economics*, 2, 193-216.

Ayong Le Kama, A.D., 2001. *Sustainable Growth, Renewable Resources and Pollution. Journal of Economic Dynamics and Control*, 25, 1911-1918.

Alvarez-Guadrado, F. and von Long, N., 2011. Consumption and Renewable Resource Extraction under Alternative Property-Rights Regimes. *Resource and Energy Economics*, 33(4), pp. 1028-1053.

Barbier, E.B., 1999. Endogenous Growth and Natural Resource Scarcity. *Environmental and Resource Economics*, 14, pp. 51-74.

Barro, R.J., 2001. Human Capital and Growth. *American Economic Review, Papers and Proceedings*, 91, pp. 12-17.

Barro, R.J. and Sala-i-Martin, X., 1995. Economic Growth. New York: McGraw-Hill, Inc.

Beltratti, A., Chichilnisky, G., and Heal, G.M., 1994. Sustainable Growth and the Golden Rule. In *The Economics of Sustainable Development*, edited by Goldin, I. and Winters, I.A. Cambridge: Cambridge University Press.

Benchekroun, H., 2003. Unilateral Production Restrictions in a Dynamic Duopoly. *Journal of Economic Theory*, 111, pp. 237-261.

Benhabib, J. and Perli, R., 1994. *Uniqueness and Indeterminacy: On the Dynamics of Endogenous Growth*. *Journal of Economic Theory*, 63, pp. 113-142.

Berck, P., 1981. Optimal Management of Renewable Resources with Growing Demand and Stock Externalities. *Journal of Environmental Economics and Management*, 11, pp.101-118.

Brander, J.A. and Taylor, M.S., 1998. The Simple Economics of Easter Island: A Ricardo-Malthus Model of Renewable Resource Use. *American Economic Review*, 81, pp. 119-138.

Brown, G.M., 2000. Renewable Natural Resource Management and Use without Markets. *Journal of Economic Literature*, 38, pp. 875-914.

Bulter, E.H. and Barbier, E.B., 2005. *Trade and Renewable Resources in a Second Best World: An Overview*. *Environmental and Resource Economics*, 30, pp. 423-463.

Bulter, E.H. and Van Kooten, G.C., 1999. Economics of Antipoaching Enforcement and the Ivory Trade Ban. *American Journal of Agricultural Economics*, 81, pp. 453-466.

Burmeister, E. and Dobell, A.R., 1970. *Mathematical Theories of Economic Growth*. London: Collier Macmillan Publishers.

Cairns, D.R. and Tian, H.L., 2010. Sustained Development of a Society with a Renewable Resource. *Journal of Economic Dynamics & Control*, 24, pp. 2048-2061.

Campbell, J.Y. and Ludvigson, S., 2001. Elasticities of Substitution in Real Business Cycle Models with Home Production. *Journal of Money, Credit and Bankings*, 33, pp. 847-875.

Castelló-Climent, A. and Hidalgo-Cabrillana, A., 2012. The Role of Education Quality and Quantity in the Process of Economic Development. Economics of Education Review, 31(4), pp. 391-409.

Chen, C.H. and Lu, Z.N., 2009. *Analysis of the Economical Growth Model with Limited Renewable Resource*. International Journal of Nonlinear Science, 7, pp. 90-94.

Chen, M.K. and Chevalier, J.A., 2008. The Taste for Leisure, Career Choice, and the Returns to Education. *Economics Letters*, 99, pp. 353-356.

Choi, S.M., 2011. How Large Are Learning Externalities? *International Economic Review*, 52, pp. 1077-1103.

Clark, C.W., 1976. *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*. New York: Wiley.

Cohn, E. and Cooper, S.T., 2004. Multi-product Cost Functions for Universities: Economies of Scale and Scope. In *International Handbook on the Economics of Education*, edited by Johnes, G. and Johnes, J., Cheltenham: Edward Elgar.

Copeland, B.R. and Taylor, M.S., 2009. Trade, Tragedy, and the Commons. *American Economic Review*, 99, pp. 725-749.

Dasgupta, P.S. and Heal, G.E., 1979. *The Economics of Exhaustible Resources*. Cambridge: Cambridge University Press.

Drugeon, J.P. and Venditti, A., 2001. Intersectoral External Effects, Multiplicities & Indeterminacies. *Journal of Economic Dynamics & Control*, 25, pp. 765-787.

Eliasson, L. and Turnovsky, S.J., 2004. Renewable Resources in an Endogenously Growing Economy: Balanced Growth and Transitional Dynamics. *Journal of Environmental Economics and Management*, 48, pp. 1018-1049.

Erceg, C., Guerrieri, L. and Gust, C., 2005. Can Long-Run Restrictions Identify Technology Shocks? *Journal of European Economic Association*, 3, pp. 1237-1278.

Farmer, K. and Bednar-Friedl, B., 2010. *Intertemporal Resource Economics – An Introduction to the Overlapping Generations Approach*. New York: Springer.

Fleisher, B., Hu, Y.F., Li, H.Z., and Kim, S.H., 2011. Economic Transition, Higher Education and Worker Productivity in China. *Journal of Development Economics*, 94, pp. 86-94.

Fujiwara, K., 2011. Losses from Competition in a Dynamic Game Model of a Renewable Resource Oligopoly. *Resource and Energy Economics*, 33, pp. 1-11.

Gordon, H.S., 1954. The Economic Theory of a Common Property Resource: The Fishery. *Journal of Political Economy*, 62, pp. 124-142.

Greenwood, J. and Hercowitz, Z., 1991. The Allocation of Capital and Time over the Business Cycle. *The Journal of Political Economy*, 99, 1188-1214.

Gylfason, T., Herbertsson, T., and Zoega, G., 1999. A Mixed Blessing: Natural Resources and Economic Growth. *Macroeconomic Dynamics*, 3, pp. 204-25.

Hahn, F.H., 1990. *Solowian Growth Models, in Growth, Productivity, Unemployment*, edited by P. Diamond. Mass., Cambridge: The MIT Press.

Hannesson, R., 2000. Renewable resources and the gains from trade. *Canadian Journal of Economics*, 33, pp. 122-132.

Hanushek, E. and Kimko, D., 2000. Schooling, Labor-Force Quality and the Growth of Nations. *American Economic Review*, 90, pp. 1194-1204.

Hussey, A., 2012. Human Capital Augmentation versus the Signaling Value of MBA. *Economics of Education Review*, 31, 442-451.

Ishida, J., 2004. Education as Advertisement. *Economics Bulletin*, 10, pp. 1-8.

Jones, L. and Manuelli, R.E., 1997. The Sources of Growth. *Journal of Economic Dynamics and Control*, 21, pp. 75-114.

Kihlstrom, R.E. and Riordan, M.H., 1984. Advertising as a Signal. *Journal of Political Economy*, 92, pp. 427-450.

Krueger, A.B., 1999. Experimental Estimates of Education Production Functions. *Quarterly Journal of Economics*, 114, pp. 497-532.

Krueger, A.B. and Lindahl, M., 2001. Education for Growth: Why and for Whom. *Journal of Economic Literature*, 39, pp. 1101-1136.

Kurz, M., 1963. The Two Sector Extension of Swan's Model of Economic Growth: The Case of No Technical Change. *International Economic Review*, 4, pp. 1-12.

Ladrón-de-Guevara, A., Ortigueira, S., and Santos, M.S., 1999. A Two Sector Model of Endogenous Growth with Leisure. *The Review of Economic Studies*, 66, pp. 609-631.

Lee, S., 2007. The Timing of Signaling: To Study in High School or in College? *International Economic Review*, 48, pp. 785-807.

Levhari, D. and Withagen, C., 1992. Optimal Management of the Growth Potential of Renewable Resources. *Journal of Economics*, 56, pp. 297-309.

Li, H.B., Liu, P.W., and Zhang, J.S., 2012. Estimating Returns to Education Using Twins in Urban China. *Journal of Development Economics*, 97, pp. 494-504.

Liu, Z.Q., 2007. The External Returns to Education: Evidence from Chinese Cities. *Journal of Urban Economics*, 61, pp. 542-564.

Long, N.V. and Wang, S., 2009. Resource-grabbing by Status-conscious Agents. *Journal of Development Economics*, 89, pp. 39-50.

Lucas, R.E., 1988. On the Mechanics of Economic Development. *Journal of Monetary Economics*, 22, pp. 3-42.

Milgrom, P. and Roberts, J., 1986. Price and Advertising Signals of Product Quality. Journal of Political Economy, 94, pp. 796-821.

Milner-Gulland, E.J. and Leader-Williams, N., 1992. A Model of Incentives for the Illegal Exploitation of Black Rhinos and Elephants. *Journal of Applied Ecology*, 29, pp. 388-401.
Munro, G.R. and Scott, A.D., 1985. The Economics of Fisheries Management. In *Handbook of Natural Resource and Energy Economics, vol. II*, edited by Kneese, A.V. and Sweeney, J.L., Amsterdam: Elsevier.
Nelson, P., 1974. Advertising as Information. *Journal of Political Economy*, 82, pp. 729-754.
Paterson, D.G. and Wilen, J.E., 1977. Depletion and Diplomacy: The North-Pacific Seal Hunt, 1880-1910. In *Research in Economic History*, edited by Uselding, P. JAI Press.
Plourde, G.C., 1970. A Simple Model of Replenishable Resource Exploitation. *American Economic Review*, 60, pp. 518-522.
Plourde, G.C., 1971. Exploitation of Common-Property Replenishable Resources. *Western Economic Journal*, 9, pp. 256-266.
Rauch, J., 1993. Productivity Gains from Geographic Concentration of Human Capital: Evidence from the Cities. *Journal of Urban Economics*, 34, pp. 380-400.
Rupert, P., Rogerson, R., and Wright, R., 1995. Estimating Substitution Elasticities in Household Production Models. *Economic Theory*, 6, 179-193.
Sachs, J.D. and Warner, A.M., 2001. The Curse of Natural Resources. *European Economic Review*, 45, pp. 827-838.
Schaefer, M.B., 1957. Some Considerations of Population Dynamics and Economics in Relation to the Management of Marine Fisheries. *Journal of Fisheries Research Board of Canada*, 14, pp. 669-681.
Solow, R., 1956. *A Contribution to the Theory of Growth. Quarterly Journal of Economics*, 70, pp. 65-94.
Solow, R., 1999. Neoclassical Growth Theory. In *Handbook of Macroeconomics*, edited by Taylor, J.B. and Woodford, M. North-Holland.
Solow, R., 2000. *Growth Theory – An Exposition*. New York: Oxford University Press.
Spence, M., 1973. Job Market Signaling. *Quarterly Journal of Economics*, 87, pp. 355-374.
Stiglitz, J.E., 1967. A Two Sector Two Class Model of Economic Growth. *Review of Economic Studies*, 34, pp. 227-238.
Stiglitz, J.E., 1974. Growth with Exhaustible Natural Resources: Efficient and Optimal Growth Paths. *Review of Economic Studies, Symposium on the Economics of Exhaustible Resources*, 123-138.
Stiglitz, J.E., 1975. The Theory of Screening, Education, and the Distribution of Income. *American Economic Review*, 65, pp. 283-300.
Tajibaeva, L.S., 2011. Property Rights, Renewable Resources and Economic Development. *Environmental and Resource Economics*, 51, pp. 23-41.
Tornell, A. and Velasco, A., 1992. The Tragedy of the Commons and Economic Growth: Why does Capital Flow from Poor to Rich Countries?, *Journal of Political Economy*, 100, 1208-1231.
Turnovsky, S.J., 1999. Fiscal Policy and Growth in a Small Open Economy with Elastic Labor Supply. *The Canadian Journal of Economics*, 32, 1191-1214.
Uzawa, H., 1961. On A Two-Sector Model of Economic Growth I. *Review of Economic Studies*, 29, pp. 47-70.
Uzawa, H., 1965. Optimal Technical Change in an Aggregative Model of Economic Growth. *International Economic Review*, 6, pp. 18-31.
Wirl, F., 2004. Sustainable Growth, Renewable Resources and Pollution: Thresholds and Cycles. *Journal of Economic Dynamics & Control*, 28, pp. 1149-1157.
Zhang, W.B., 1993. Woman's Labor Participation and Economic Growth - Creativity, Knowledge Utilization and Family Preference. *Economics Letters*, 42, pp. 105-110.
Zhang, W.B., 2007. Economic Growth with Learning by Producing, Learning by Education, and Learning by Consuming. *Interdisciplinary Description of Complex Systems*, 5, pp. 21-38.
Zhang, W.B., 2011. Renewable Resources, Capital Accumulation, and Economic Growth. *Business Systems Research*, 1, pp. 24-35.

7. Appendix: Proving Lemma 1

The appendix shows that the dynamics can be expressed by three dimensional differential equations. From (2), (5) and (7), we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_e N_e}{K_e} = \frac{\tilde{\alpha}_x N_x}{K_x}, \tag{A1}$$

where we omit time index and $\tilde{\alpha}_j \equiv \dfrac{\alpha_j}{\beta_j}, \ j = i, e, x.$

By (1), we have

$$f_i(z, x_i) \equiv \frac{F_i}{N_i} = A_i x_i^{\gamma_i} \left(\frac{\tilde{\alpha}_i}{z}\right)^{\alpha_i}, \tag{A2}$$

where we also use (A1) and $x_i \equiv X_i / N_i$. By (2), (A1) and (A2), we have

$$r + \delta_k = \frac{\alpha_i z f_i}{\tilde{\alpha}_i}, \ w = \beta_i f_i, \ p_x = \frac{\gamma_i f_i}{x_i}. \tag{A3}$$

We can express w, r and p_x as functions of z and x_i. From (5) and (A1), we solve

$$r + \delta_k = \frac{\alpha_x p_x A_x X^b z^{\beta_x}}{\tilde{\alpha}_x^{\beta_x}}, \ w = \frac{\beta_x p_x A_x X^b \tilde{\alpha}_x^{\alpha_x}}{z^{\alpha_x}}. \tag{A4}$$

From (A2) and the marginal conditions for labor in (A3) and (A4), we have

$$p_x = \frac{\beta_0 x_i^{\gamma_i} z^{\alpha_x - \alpha_i}}{X^b}, \tag{A5}$$

where $\beta_0 \equiv \dfrac{\beta_i A_i \tilde{\alpha}_i^{\alpha_i}}{\beta_x A_x \tilde{\alpha}_x^{\alpha_x}}$.

From (A5) and (A3), we solve

$$x_i = \frac{\gamma_i \beta_x A_x \tilde{\alpha}_x^{\alpha_x}}{\beta_i} \frac{X^b}{z^{\alpha_x}}. \tag{A6}$$

From the above analyses, we express w, f_i, r, p_x and x_i as functions of z and X. From (6) and (7), we have

$$p_e = \frac{w z^{\alpha_e}}{\beta_e A_e \tilde{\alpha}_e^{\alpha_e}}. \tag{A7}$$

where we also use (A1). Hence, we can express p_e and $\bar{p}$ as functions of z, X, and H.

From (20), (2) and $p_x c_x = \chi \bar{y}$ in (14), we get $\chi \bar{N} \bar{y} + \gamma_i F_i = p_x F_x$.

From $T_e = \eta \bar{y} / \bar{p}$ in (14) and (16), we have $\bar{y} = \dfrac{\bar{p} F_e}{\bar{N} \eta}$.

Insert this equation in $\chi \bar{N} \bar{y} + \gamma_i F_i = p_x F_x$

$$\frac{\chi \bar{p} F_e}{\eta} + \gamma_i F_i = p_x F_x . \tag{A8}$$

From (2), (5) and (7), we have

$$r + \delta_k = \frac{\alpha_i F_i}{K_i} = \frac{\alpha_x p_x F_x}{K_x} = \frac{\alpha_e p_e F_e}{K_e}. \tag{A9}$$

Substituting (A9) into (A8) yields

$$K_x = \bar{p}_e K_e + \bar{\gamma}_i K_i , \tag{A10}$$

where $\bar{p}_e(z, X, H) \equiv \frac{\alpha_x \chi \bar{p}}{\eta \alpha_e p_e}$, $\bar{\gamma}_i \equiv \frac{\alpha_x \gamma_i}{\alpha_i}$,

Insert (A1) in $N_i + N_e + N_x = N$

$$\frac{K_i}{\tilde{\alpha}_i} + \frac{K_e}{\tilde{\alpha}_e} + \frac{K_x}{\tilde{\alpha}_x} = \frac{N}{z}. \tag{A11}$$

Insert (A10) in $K_i + K_e + K_x = K$ and (A11)

$$\varepsilon_{1i} K_i + \varepsilon_{1e} K_e = \frac{N}{z},$$
$$\varepsilon_{2i} K_i + \varepsilon_{2e} K_e = K, \tag{A12}$$

where $\varepsilon_{1i} \equiv \frac{1}{\tilde{\alpha}_i} + \frac{\bar{\gamma}_i}{\tilde{\alpha}_x}$, $\varepsilon_{1e}(z, X, H) \equiv \frac{1}{\tilde{\alpha}_e} + \frac{\bar{p}_e}{\tilde{\alpha}_x}$, $\varepsilon_{2i} \equiv 1 + \bar{\gamma}_i$, $\varepsilon_{2e}(z, X, H) \equiv 1 + \bar{p}_e$.

Solve (A12) with K_i and K_e as variables

$$K_i = \left(\frac{\varepsilon_{2e} N}{z} - \varepsilon_{1e} \bar{k} \bar{N} \right) \varepsilon, \quad K_e = \left(\varepsilon_{1i} \bar{k} \bar{N} - \frac{\varepsilon_{2i} N}{z} \right) \varepsilon, \tag{A13}$$

where we use $K = \bar{k} \bar{N}$ and $\varepsilon(z, X, H) \equiv \frac{1}{\varepsilon_{1i} \varepsilon_{2e} - \varepsilon_{2i} \varepsilon_{1e}}$.

By (A13) and (A10), we solve the capital distribution, K_i, K_e and K_x, as functions of z, X, N, H, and $\bar{k}$. By (A1), we solve the labor distribution, N_i, N_e and N_x, as functions of z, X, N, H, and $\bar{k}$.

From (2), (5) and (7), we have

$$F_i = \frac{w N_i}{\beta_i}, \quad X_i = \frac{\gamma_i F_i}{p_x}, \quad F_x = \frac{w N_x}{\beta_x p_x}, \quad F_e = \frac{w N_e}{\beta_x p_e}. \tag{A14}$$

We express F_i, X_i, F_x, and F_e as functions of z, X, N, and $\bar{k}$. From (7) and $\bar{y} = \bar{p} F_e / \bar{N} \eta$, we have

$$\bar{y} = p_w N_e, \tag{A15}$$

where $p_w(z, X, H) \equiv \dfrac{\bar{p} w}{\beta_e \bar{N} \eta p_e}$.

From (A15) and $p_x c_x = \chi \bar{y}$ in (14), we express $\bar{y}$ and c_x as functions of z, X, N, H, and $\bar{k}$. From $w H^m T_h = \sigma \bar{y}$, $\bar{p} T_e = \eta \bar{y}$, and the definition of $\bar{y}$, we have

$$T_h = \left(\frac{1+r}{H^m w} \right) \sigma \bar{k} + \sigma T_0,$$

$$T_e = \frac{(1+r)\eta \bar{k}}{\bar{p}} + \frac{T_0 \eta H^m w}{\bar{p}}. \tag{A16}$$

From $T + T_e + T_h = T_0$ and (A16), we have

$$T = h_1 - h_2 \bar{k}, \tag{A17}$$

where $h_1(z, X, H) \equiv T_0 - \sigma T_0 - \dfrac{T_0 \eta H^m w}{\bar{p}}$, $h_2(z, X, H) \equiv \left(\dfrac{\sigma}{H^m w} + \dfrac{\eta}{\bar{p}} \right)(1+r)$.

From $N = H^m T \bar{N}$, and (A17), we have

$$N = \left(h_1 - h_2 \bar{k}\right) \bar{N} H^m. \tag{A18}$$

From (19) and $K = \bar{k} \bar{N}$, we have

$$(\xi + \lambda) \bar{y} - \delta \bar{k} = \frac{w N_i}{\beta_i \bar{N}}, \tag{A19}$$

where we also use (14) and (A14). Substituting the definition of $\bar{y}$ and $N_i = z K_i / \tilde{\alpha}_i$ into (A19) yields

$$\left[\tilde{\xi}(1+r) - \delta\right]\bar{k} + \tilde{\xi} T_0 H^m w = \frac{\tilde{\varepsilon}_2 N}{z} - \tilde{\varepsilon}_1 \bar{k} \bar{N},$$

where we also use (A13) and $\tilde{\xi} \equiv \xi + \lambda$, $\tilde{\varepsilon}_j(z, X, H) \equiv \dfrac{\varepsilon \varepsilon_{je} z w}{\beta_i \tilde{\alpha}_i \bar{N}}$, $j = 1, 2$.

Insert (A18) into the above equation

$$\bar{k} = \Psi(z, X, H) \equiv H^m \left[\frac{\tilde{\varepsilon}_2 h_1 \bar{N}}{z} - \tilde{\xi} T_0 w \right] \left[\tilde{\xi}(1+r) + \tilde{\varepsilon}_1 \bar{N} + \frac{\tilde{\varepsilon}_2 h_2 \bar{N} H^m}{z} - \delta \right]^{-1}. \tag{A20}$$

From (A20), we solve $\bar{k}$ as a function of z, X and H.

It is straightforward to check that all the variables can be expressed as functions of z, X and H at any point of time by the following procedure: x_i by (A6) $\rightarrow p_x$ by (A5) $\rightarrow r$ by (A3) $\rightarrow w$ by (A3) $\rightarrow \bar{k}$ by (A20) $\rightarrow K = \bar{k}\,\bar{N} \rightarrow N$ by (A18) $\rightarrow T = N / H^m \bar{N} \rightarrow T_h$ and T_e by (A16) $\rightarrow p_w$ by the definition $\rightarrow p_e$ by (A16) $\rightarrow K_i$ and K_e by (A13) $\rightarrow K_x$ by (A11) $\rightarrow N_i$, N_e, and N_x by (A1) $\rightarrow \bar{y}$ by (A15) $\rightarrow c_x$, c, s by (14) $\rightarrow X_i = x_i N_i \rightarrow F_i$ by (1) $\rightarrow F_x$ by (4) $\rightarrow F_e$ by (6) $\rightarrow U$ by (11).

We note that the right-hand sides of (3) and (8) are functions of z, X and H. Hence, we have

$$\dot{X}(t) = \Lambda_X(z, X, H),$$
$$\dot{H}(t) = \Lambda_H(z, X, H), \tag{A21}$$

where we do explicitly express Λ_X and Λ_H as it straightforward but their expressions are tedious.

Taking derivatives of (A20) with respect to t yields

$$\dot{\bar{k}} = \frac{\partial \Psi}{\partial z}\dot{z} + \Lambda_X \frac{\partial \Psi}{\partial X} + \Lambda_H \frac{\partial \Psi}{\partial H}, \tag{A22}$$

where we also use (A21). From (15), we have

$$\dot{\bar{k}} = \lambda \bar{y}(z, X, H) - \bar{k}. \tag{A23}$$

From (A22) and (A23), we solve

$$\dot{z} = \Lambda_z(z, X, H) \equiv \left(\lambda \bar{y} - \Psi - \Lambda_X \frac{\partial \Psi}{\partial X} - \Lambda_H \frac{\partial \Psi}{\partial H}\right)\left(\frac{\partial \Psi}{\partial z}\right)^{-1}. \tag{A24}$$

We thus proved the lemma.