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Pretension Strategy in the Surviving Game

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Presently, we cannot find the scientific analysis that clearly explains the deepest roots of global economical and moral crisis. Because of that many famous politicians, economists, sociologists denote the understanding of current situation as the most valuable attainment. Under traditional influence of the doctrine of spontaneous harmony of egoistic individual behavior many economists believe that competition and private property rights through the markets' price mechanism leads in the long run to the Pareto efficiency equilibrium. At the same time the social and economic reality categorically asks to ascertain the market failure and to revision the classical statements of microeconomics. The perfect competition market has lost its attributes due to dialectics of interactions of agents. The investigation of the interactions strategies of the individuals are based on the game theory, what also helps to understand the role of asymmetric information as a specific market failure factor. In the present paper the Martin Shubik classical surviving game is analyzed and some statements of Herbert Gintis concerning this game are critically appraised. The solution of Martin Shubik game in the original geometrical form is offered. The problem of Martin Shubik "does the fittest necessary survive?" is transformed according the case of asymmetric information in problem "does the pretender survive?", for which the answer "if the agent is not the weakest, but he pretends to be the weakest, than this agent survives with high probability" is offered. The results of the present paper appear to be innovative, not discussed in literature available to the author of the present paper.

Keywords*: game, probability of surviving, interactions of the agent's strategies, Nash equilibrium, asymmetric information, pretension*

JEL Classification: C70, C73

1. Introduction

The view about wide diversity of game theory applications offers, for instance, the content of the Gintis's (2009, p.390) book: "*Game Theory Evolving: A Problem Centered Introduction to Modelling Strategic Interaction.*"

The applications of game theory as tool for decision support is given in the books of Baye (1997, p. 578), Байе (1999, p. 743), Binmore (2007, p. 184), Jaunzems (2008, p. 555; 2009a, p.311; 2009b, p. 360). In paper of Jaunzems (2009c) the analysis of some social economical processes in Latvia with help of game theory is presented. There is no lack of literature and sources, nevertheless in the practice of social and economic analysis in Latvia the game theory introduces too slowly.

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In Gintis's book (2009, p. 23) under the title "*When Weakness Is Strength*" the model "The surviving game of three agents" offered by game theorist *Martin Shubik* (1954, 43-46) is considered. With help of this model the surviving example which contradicts to the fundamental thesis of Charles Darvin "*survival of the fittest*" is constructed. In this example the highest probability to survive has the weakest player. However this example is too simplified. The deepest investigation of the problem discovers that probability of surviving depends on mutual proportion of agents' strength and from behavior of agents as well. Unfortunately the author of present paper was not able to get original paper of Martin Shubik (Shubik, 1954, pp. 43-46). Therefore the aim of present paper is dispute with Herbert Gintis (2009) and more wide interpretation of Charles Darvin thesis. The limitation of some statements of Herbert Gintis (2009) empirically is proved. In this paper exact definition of surviving game is given, the mathematical model of surviving game under definite comparatively universal assumptions is constructed and with help of model multi-shaped numerical experiments are performed. Besides that the Nash equilibrium of surviving game is identified, the classical surviving game is modified according asymmetric information and instead the theme "*When Weakness Is Strength*" the theme "*When Pretension Is Strength*" is discussed. The authors have never met the mentioned results in literature, therefore comments are welcomed and, if is not proved the opposite, results of the present paper have to be evaluated as innovative.

2. The Definition of the Surviving Game

The definition of the surviving game comes next, with comments.

Three individuals − Ansis, Basis, Casis take part in the surviving game.

At first the sequence of shooting during lottery is determined. As result one from six possible sequences (ABC), (ACB), (BAC), (BCA), (CAB), (CBA) is fixed. The probability of each sequence is 1:6.

The game is extensive − in the each step or act of the game one of players has rights to shot.

Explanation. Let us consider, for instance, the sequence (ABC). The sequence (ABC) means, that Ansis shot the first. Ansis has three strategies: Ansis may shot Basis, Ansis may shot Casis and Ansis may shot in the air.

(1) Let us assume that Ansis shot Basis; we will denote that as A→B. If Ansis hits Basis than Basis exits the game (we will say Basis is shot down or Basis is eliminated) and farther sequence of shooting is (CA). If Ansis miss Basis then the farther sequence of shooting is (BCA).

(2) Let us assume that Ansis shot Casis; we will denote that as A→C. If Ansis hits Casis than Casis exits the game, he is eliminated and farther sequence of shooting is (BA). If Ansis miss Casis then the farther sequence of shooting is (BCA).

(3) Let us assume that Ansis shot in the air; we will denote that as A→O. The farther sequence of shooting is (BCA).

The five another sequences are interpreted analogically.

The definition of surviving game continued. Let us assume that the following probabilities are given.

The probability that Ansis hits is p_A. The probability that Ansis miss we denote as q_A: $p_A + q_A = 1$.

The probability that Basis hits is p_B. The probability that Basis miss we denote as q_B: $p_B + q_B = 1$.

The probability that Casis hits is p_C. The probability that Casis miss we denote as q_C: $p_C + q_C = 1$.

The each of agents is interested to stay alive. The game is finished when only one of the players has left alive. Let us observe that the surviving game may have infinite acts because of possibility that players miss and miss and all three agents or at least two of them have infinite chances of shooting. In the each act the agent, who has the move, chooses the strategy from set of three strategies, if three players are still alive, or only one strategy if two players are remain.

Our main goal is the determination of the Nash equilibrium of this game.

Let us illustrate the Nash equilibrium in the simple, but nevertheless a very pithy case.

Suppose $p_A = 1$, $p_B = 1$, $p_C = 1$ and the sequence (ABC) during lottery is determined. It is easy to see that the triple of strategies (A→O, B→O, C→O) determines a situation that is very stable Nash equilibrium. Indeed, if Ansis shot Basis, Basis is eliminated and the next moves Casis who shots down Ansis and wins. So, Ansis would be unwise if he shots Basis or Casis. The best strategy for Ansis is shot in the air. When the right of move has Basis he judges similarly and makes decision that shot in the air is the best strategy for him. The same decision makes Casis. So the eternal peace exists between these three antagonistic and strong warriours.

The given surviving game associates with popular but wrong thesis about bipolar world as peace guarantee. If only two players Ansis and Basis take part in surviving game and $p_A = 1$, $p_B = 1$, than shooter who has rights to shot first unswerving eliminates the enemy. We recognize here the first move advantage often discussed by military experts.

Conclusion. The antagonistic bipolar world is unstable. The antagonistic tripolar world is very stable.

Important remark. Everywhere further, if especially is not formulated another assumption, we will examine comparatively universal case, namely, we will suppose that $0 < p_C < p_B < p_A \leq 1$. We will say that Ansis is the strongest but Casis is the weakest shooter.

What is the payoff of the player? The payoff of the player could be the surviving probability. However, if we are going to investigate this game analytically with help of mathematical methods we need correct agents' surviving probability definition, what unfortunately we do not meet in Gintis's book (2009) published by Princeton University Press.

Let us illustrate with help of simplest stochastic experiment − dice throw the theoretical difficulties what arise when we are going to definite agents' surviving probability. Try to understand the meaning of following question: "Is the probability that first will come in sight even number bigger than the probability that first will come in sight number one?"

It is possible to calculate, for instance, probability of event "five times throwing the dice the even number will come in sight earlier than number one". In the same time the cardinality of the set of all outcomes when the first comes in sight even number is continuum. Also the cardinality of the set of all outcomes when the first comes number one is continuum.

In order to depict the extensive process of the surviving game graphically as decision-state tree we have to use infinite graph with infinite volume of infinite sub-branches.

2.1. Two Agents' Surviving Game

Before investigation of three agents' surviving game is purposeful to start with two agents' surviving game analysis, what is sufficiently simpler because of each player has only one strategy − to shot enemy.

For instance, let us examine the case when Casis is eliminated and further sequence is (AB). Such a game is possible to depict geometrically as decision-state tree with infinite volume of finite sub-branches and one infinite subbranche.

We are going to define the probability that Ansis survives as sum of infinite geometric series. Let us denote probability that Ansis has eliminated Basis after 2k+1 shoots or earlier with $P_A\{(AB) \mid 2k+1\}$. Let the sign "+" means "hits", the sign "−" means "miss".

Then, for instance, in the case 2k+1 = 5 the event we are interested in consists from the following results of shooting: (A+), (A−, B−, A+), (A−, B−, A−, B−, A+).

We calculate $P_A\{(AB) \mid 5\}$ as follows:

$$P_A\{(AB) \mid 5\} = p_A + q_A q_B p_A + (q_A q_B)^2 p_A = p_A [1 + q_A q_B + (q_A q_B)^2].$$

It is easy to see by analogy that $P_A\{(AB) \mid 2k+1\} = p_A [1 + q_A q_B + ... + (q_A q_B)^k]$.

We calculate the limit of partial sum of the geometric series:

$$P_A(AB) := \lim_{k\to\infty} P_A\{(AB) \mid 2k+1\} = \frac{p_A}{1 - q_A q_B}.$$

Number $P_A(AB)$ will be interpreted as probability that Ansis survives.

The probability $P_B(AB)$ is defined analogically: $P_B(AB) := \lim_{k\to\infty} P_B\{(AB) \mid 2k\} = \frac{q_A p_B}{1 - q_A q_B}$.

As a check on our formulas, note that $P_A(AB) + P_B(AB) = 1$.

Remark. In book published in Princeton University (Gintis, 2009) the probability $P_A(AB)$ has calculated from the recursion equation: $P_A(AB) = p_A + q_A P_A(BA) = p_A + q_A q_B P_A(AB)$.

Solving, we get $P_A(AB) = \frac{p_A}{1 - q_A q_B}$.

Let us mark that this method is heuristic but mathematically incorrect.

In the analysis of three agents' surviving game we will make use of the following theorem about two agents' game what confirms the intuitive suspected connection: the weakest enemy of Ansis, the higher is for Ansis probability to survive.

Theorem. $P_A(AB) < P_A(AC)$ if and only if $p_B > p_C$.

The proof follows from equalities $P_A(AB) = \dfrac{p_A}{1-q_Aq_B}$; $P_A(AC) = \dfrac{p_A}{1-q_Aq_C}$.

Consequence. $P_A(BA) < P_A(CA)$ if and only if $p_B > p_C$.

2.2. The Nash Equilibrium of the Sub-Game of the Surviving Game and Nash Equilibrium of the Martin Shubik Game

As it was showed before in general case of surviving game the cardinality of agents' strategies set is continuum. We are going to make important assumption about agents' behavior in three shooters game, which will relieve us the analysis of game and will allow us to get important and pithy interpreted conclusions.

Let us suppose that each of agents chooses his strategy in the very beginning of the game and after that each time when agent has right to move, he acts according this strategy. It means, that Ansis in the very beginning of the game chooses one of three strategies A→B, A→C, A→O, and then utilizes this strategy each time when he has rights to move (naturally, till there are three players in the game). Analogically, Basis chooses one of strategies B→A, B→C, B→O and Casis chooses one of strategies C→A, C→B, C→O.

Taking in account this assumption all together 3×3×3 = 27 triples of strategies exist.

Concrete triple of strategies, for instance, (A→B; B→A; C→O) allows us to follow the extensive process of game so say in the probabilities tongue and analogically as it was did in the case of two players to define agents' surviving probabilities as sum of geometric series.

Given sequence of acts (ABC) and given triple of strategies (A→B; B→A; C→O) determine three surviving probabilities: $P_A(ABC)$, $P_B(ABC)$ $P_C(ABC)$.

The game what corresponds to the concrete sequence determined during lottery we will call as sub-game of the surviving game. We have six sub-games all together, each of that corresponds to one of six possible shooting sequences (ABC), (ACB), ... , (CBA).

The Martin Shubik game is completely defined, if we construct the (27×3)-table represented strategy-surviving probabilities for all sub-games. Thus, in order to investigate Martin Shubik game we calculate 6×27×3 = 486 probabilities. After that, taking in account that initial sequences are stochastic, we calculate the (27×3)-table of mathematical expectations of strategy-surviving probabilities. These table allows us to determine Nash equilibrium of Martin Shubik game and to make different another conclusions.

Let us utilize heiristic method of surviving probabilities determination from the recursion equations offered by Herbert Gintis. (The same results can be get as limits of geometric series.) Than, for instance, we will get the following expressions for Ansis, Basis and Casis surviving probabilities in the sub-game (ABC) concerning strategies triple N := (A→B; B→A; C→O):

$P_A^N(ABC) = p_A\, P_A(CA) + q_A\, P_A^N(BCA) = p_A\, P_A(CA) + q_A\, q_B\, P_A^N(CAB) =$

$= p_A\, P_A(CA) + q_A\, q_B\, P_A^N(ABC)$. Solving, we get $P_A^N(ABC) = \dfrac{p_A P_A(CA)}{1-q_Aq_B}$.

$P_B^N(ABC) = q_A\, P_B^N(BCA) = q_A\, [p_B\, P_B(CB) + q_B\, P_B^N(CAB)] =$

$= q_A\, p_B\, P_B(CB) + q_A\, q_B\, P_B^N(ABC)]$. Solving, we get $P_B^N(ABC) = \dfrac{q_Ap_BP_B(CB)}{1-q_Aq_B}$.

$P_C^N(ABC) = p_A\, P_C(CA) + q_A\, P_C^N(BCA) = p_A\, P_C(CA) + q_A\, [p_B\, P_C(CB) + q_B\, P_C^N(CAB)] =$

$= p_A\, P_C(CA) + q_A\, p_B\, P_C(CB) + q_A\, q_B\, P_C^N(ABC)$.

Solving, we get $P_C^N(ABC) = \dfrac{p_AP_C(CA)+q_Ap_BP_C(CB)}{1-q_Aq_B}$.

As a check on our formulas, note that $P_A^N(ABC) + P_B^N(ABC) + P_C^N(ABC) =$

$$= \frac{p_AP_A(CA)}{1-q_Aq_B} + \frac{q_Ap_BP_B(CB)}{1-q_Aq_B} + \frac{p_AP_C(CA)+q_Ap_BP_C(CB)}{1-q_Aq_B} = 1$$

Remark. The symbols P_A^N (ABC), P_B^N (ABC), P_C^N (ABC) reflect the sub-game (ABC) and strategies triple N = (A→B; B→A; C→O) choosen as well.

What strategy does Ansis choose? Our intuition predict us that Ansis will try to eliminate the most dangerous enemy, namely, the enemy with higher hit probability. That is Basis. In Basis turn purposeful is eliminate Ansis. For the weakest shooter Casis purposeful is shot in the air in order the strongest enemies shot each other.

Let us stress that just told mathematically is incorrect. That is typically for game theory: correct are solely statements inside the frame of mathematical concepts, any other debate as usually is defective and leads to the wrong conclusions. By my opinion directly that creates objective difficulties for the wide applications of the game theory in the research of social-economical processes.

The next lemma has universal character.

Lemma. Conditional Nash equilibrium of the sub-game.

Let us assume, that $0 < p_C < p_B < p_A \leq 1$; suppose that we have freely chosen some sub-game. Let us assume that Casis as strategy leader decides shot in the air. Than situation associated with strategy triple N := (A→B; B→A; C→O) is conditional Nash equilibrium.

Let us mark the idea of proof.

Let us suppose that sub-game is determined by sequence (ABC) and Casis chooses strategy C→O. We will prove that situation N = (A→B; B→A; C→O) is the conditional Nash equilibrium of the sub-game with corresponding payoffs P_A^N (ABC), P_B^N (ABC), P_C^N (ABC).

The proof for each sub-game consists from two steps.

First, we make certain that in the situations S := (A→C; B→A; C→O), T := (A→O; B→A; C→O) probability for Ansis to survive decreases to compare with probability P_A^N (ABC).

Let us calculate:

$$P_A^S(ABC) = p_A\, P_A(BA) + q_A\, P_A^S(BCA) = p_A\, P_A(BA) + q_A\, q_B\, P_A^S(CAB) =$$
$$= p_A\, P_A(BA) + q_A\, q_B\, P_A^S(ABC), \text{ Solving, we get } P_A^S(ABC) = \frac{p_A P_A(BA)}{1-q_A q_B}.$$

Comparing probability P_A^S (ABC) with probability P_A^N (ABC) getting before, and taking in acount that $P_A(BA) < P_A(CA)$, we get $P_A^S(ABC) < P_A^N(ABC)$.

Comparing probability P_A^T (ABC) with probability P_A^N (ABC) is easier:
from $P_A^T(ABC) = P_A^T(BCA) = q_B\, P_A^T(CAB) = q_B\, P_A^T(ABC)$ follows that $P_A^T(ABC) = 0$.

In the second step we make certain that in the situations U := (A→B; B→C; C→O), V := (A→B; B→O; C→O) probability for Basis to survive decreases to compare with probability P_B^N (ABC).

Let us calculate:

$$P_B^U(ABC) = q_A\, P_B^U(BCA) = q_A\, p_B\, P_B(AB) + q_A\, q_B\, P_B^U(CAB) =$$
$$= q_A\, p_B\, P_B(AB) + q_A\, q_B\, P_B^U(ABC). \text{ Solving, we get } P_B^U(ABC) = \frac{q_A p_B P_B(AB)}{1-q_A q_B}.$$

Comparing probability P_B^U (ABC) with probability P_B^N (ABC) getting before, and taking in acount that $P_B(AB) < P_B(CB)$, we get $P_B^U(ABC) < P_B^N(ABC)$.

We have also $P_B^V(ABC) = q_A\, P_B^V(BCA) = q_A\, P_B^V(ABC)$, from what follow $P_B^V(ABC) = 0$.

It may be prooved analogically, that situation associated with strategy triple N = (A→B; B→A; C→O) is conditional Nash equilibrium in other five sub-games.

Consequence. The conditional Nash equilibrium in Martin Shubik game. Let us asume that $0 < p_C < p_B < p_A \leq 1$. Let Casis is strategy leader and chooses strategy C→O. Then situation associated with strategy triple N = (A→B; B→A; C→O) is conditional Nash equilibrium in Martin Shubik game.

Although it is not assert in explicit form the content of theme 1.32 "*When Weakness Is Strength*" in Gintis (2009) creates impression, that triple of strategies N = (A→B; B→A; C→O) determines Nash equilibrium in the each sub-game and in the Martin Shubik game. In the example examined in details this statement is true. Multishaped numerical experiments also signalized that this statement could be true. However the aspirations to prove this statement mathematically leads to the conclusion that for some triple of probabilities p_A, p_B, p_C situation associated with triple of strategies N = (A→B; B→A; C→O) is not the Nash equilibrium.

Let us examine, for instance, the sub-game (ABC). Taking in account the proof of lemma it would be proved that strategy triple N = (A→B; B→A; C→O) determines the Nash equilibrium in the sub-game (ABC), if we were be able to prove that in the situations W := (A→B; B→A; C→A), Y := (A→B; B→A; C→B) the payoff of Casis decreases to compare with P_C^N (ABC).

It is possible to get the following expression:

$$P_C^W(ABC) = p_A P_C(CA) + q_A P_C^W(BCA) = p_A P_C(CA) + q_A [p_B P_C(CB) + q_B P_C^W(CAB)] =$$
$$= p_A P_C(CA) + q_A p_B P_C(CB) + q_A q_B [p_C P_C(BC) + q_C P_C^W(ABC)].$$

Solving, we get $P_C^W(ABC) = \dfrac{p_A P_C(CA) + q_A p_B P_C(CB) + q_A q_B p_C P_C(BC)}{1 - q_A q_B q_C}$.

Before we have got the expression $P_C^N(ABC) = \dfrac{p_A P_C(CA) + q_A p_B P_C(CB)}{1 - q_A q_B}$.

We are interested in investigation of the the set $\{(p_A, p_B, p_C)\}$ of inequality $P_C^W(ABC) < P_C^N(ABC)$ solutions and in the set $\{(p_A, p_B, p_C)\}$ of inequality $P_C^Y(ABC) < P_C^N(ABC)$ solutions. The problem arises: how to haracterize the sets of solution of inequalities $P_C^W(ABC) < P_C^N(ABC)$, $P_C^Y(ABC) < P_C^N(ABC)$.
We are interested also in investigation of the the sets of analogical inequality solutions for the other five sub-games in order to discover conditions what guarantee that strategy triple N = (A→B; B→A; C→O) determines the Nash equilibrium.

I could not receive explicit algebraic description for these solution sets. However I have received empirical descriptions of the inequality $P_C^W(ABC) < P_C^N(ABC)$, $P_C^Y(ABC) < P_C^N(ABC)$ solution sets, displayed in the table 1 and in the figure 1. In the table 1 information about minimal value of probability p_C (depending of values p_A, p_B) is given; with this probability or bigger probability in the situation, determined by strategy triple N = (A→B; B→A; C→O), Casis does not refuse the strategy C→O. If, in addition $p_C < min\{p_A, p_B\}$, then strategy triple N = (A→B; B→A; C→O) determines the Nash equilibrium of the Martin Shubik game.

In the figure 1 dates of the table 1 are depicted graphically. For each value of probability p_A, namely, $p_A = 0,1$; $p_A = 0,2$; $p_A = 0,3$; $p_A = 0,4$; $p_A = 0,5$; $p_A = 0,6$; $p_A = 0,7$; $p_A = 0,8$; $p_A = 0,9$, $p_A = 1$ corresponds curve, what show connection between probabilities p_B, p_C, holding value of p_A constant. Higher the probability p_A, higher is dislocated corresponding curve.
The figure 1 could be considered as empirical solution of the Martin Shubik game.

Table 1. *Information about Nash equilibrium conditions in Martin Shubik game.*

p_A	p_B	*min* p_C	p_A	p_B	*min* p_C
0,1	0,02	0,7754	0,6	0,5	0,0431

0,1	0,04	0,5041	0,6	0,6	0,0409
0,1	0,06	0,2501	0,7	0,05	0,9242
0,1	0,08	0,0561	0,7	0,1	0,8310
0,1	0,085	0,0326	0,7	0,2	0,5894
0,1	0,09	0,0227	0,7	0,3	0,2653
0,2	0,02	0,8954	0,7	0,4	0,0721
0,2	0,05	0,6973	0,7	0,5	0,0512
0,2	0,10	0,3309	0,7	0,6	0,0462
0,2	0,12	0,1925	0,7	0,7	0,0448
0,2	0,15	0,0483	0,8	0,05	0,9345
0,2	0,18	0,0257	0,8	0,1	0,8549
0,2	0,19	0,0237	0,8	0,2	0,6506
0,3	0,02	0,9325	0,8	0,3	0,3698
0,3	0,05	0,8073	0,8	0,4	0,1059
0,3	0,10	0,5596	0,8	0,5	0,0617
0,3	0,15	0,2874	0,8	0,6	0,0524
0,3	0,20	0,0673	0,8	0,7	0,0493
0,3	0,23	0,0385	0,8	0,8	0,0485
0,3	0,25	0,0328	0,9	0,05	0,9424
0,3	0,28	0,0289	0,9	0,1	0,8732
0,4	0,02	0,9503	0,9	0,2	0,6977
0,4	0,05	0,8600	0,9	0,3	0,4560
0,4	0,10	0,6804	0,9	0,4	0,1666
0,4	0,15	0,4695	0,9	0,5	0,0762
0,4	0,20	0,2407	0,9	0,6	0,0602
0,4	0,25	0,0714	0,9	0,7	0,0546
0,4	0,30	0,0413	0,9	0,8	0,0525
0,4	0,35	0,0347	0,9	0,9	0,0519
0,5	0,02	0,9608	1	0,05	0,9488
0,5	0,05	0,8905	1	0,1	0,8876
0,5	0,1	0,7520	1	0,2	0,7350
0,5	0,2	0,3959	1	0,3	0,5265
0,5	0,3	0,0714	1	0,4	0,2516
0,5	0,4	0,0408	1	0,5	0,0973
0,6	0,05	0,9103	1	0,6	0,0699
0,6	0,1	0,7985	1	0,7	0,0610
0,6	0,2	0,5078	1	0,8	0,0570
0,6	0,3	0,1501	1	0,9	0,0553
0,6	0,4	0,0530	1	1	0,0548

The explanation of the table 1 and figure 1.

For instance, we can read in table 1 and observe visually in figure 1: if $p_A = 0,8$; $p_B = 0,4$, then for each probability p_C bigger then 0,1059 Casis shots in the air. Let us remember, that in Martin Shubik game Ansis shots Basis and Basis shots Ansis because of $p_C < p_B$, $p_C < p_A$, namely, Casis is the weakest. Therefore, if probability p_C satisfates inequality $0,1059 < p_C < min\{p_A, p_B\} = 0,4$, then the strategy triple N = (A→B; B→A; C→O) determines the Martin Shubik games' Nash equilibrium. If $p_C < 0,1059$, then strategy triple N = (A→B; B→A; C→O) does not determine the Nash equilibrium because of Casis shots Ansis.

If $p_C > 0,4$, then Casis shots in the air if conditions A→B; B→A fulfil. But Casis is now stronger than Basis. What reason in this case is for Ansis to shot Basis?

*Another exam*ple. We can read in table 1 and observe visually in figure 1: if $p_A = 0,5$; $p_B = 0,2$, then for each probability p_C bigger then 0,3959 Casis shots in the air. However with such probability the conditions of Martin Shubik game does not fulfil. If probabilities follow in order $p_B < p_C < p_A$, then in Martin Shubik game Ansis shots Casis not Basis, because of Basis is weakest. But just $p_C < 0,2$ immediately Casis shots

Ansis. Obviously, in the case $p_A = 0{,}5$; $p_B = 0{,}2$ such probability $p_C < min\{p_A, p_B\}$ that strategy triple (A→B; B→A; C→O) determines Martin Shubik games' Nash equilibrium does not exist.

In figure 1 especially is depicted the point $(p_B, p_C) = (0{,}8; 0{,}5)$ in order to illustrate the very special case $(p_A, p_B, p_C) = (1; 0{,}8; 0{,}5)$ examined by Herbert Gintis (2009). Let us mark, that this very special case does not expose the complication of surviving game. Our calculations and figure 1 shows: still the probability p_C satisfies inequalities $0{,}0570 < p_C < 0{,}8$ strategy triple (A→B; B→A; C→O) determines Nash equilibrium, namely, Casis shots in the air. In the same time if, for instance, $(p_A, p_B, p_C) = (1; 0{,}8; 0{,}05)$ then Casis shots Ansis not in the air.

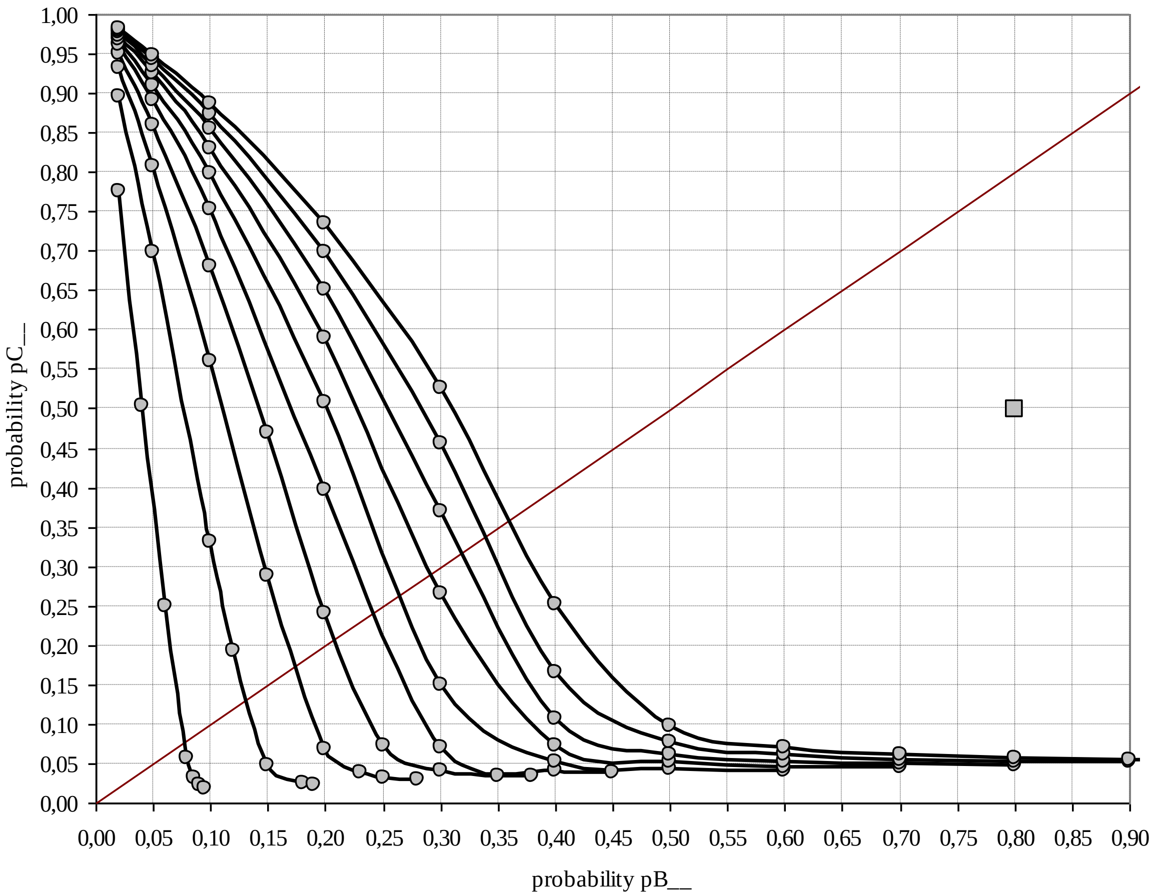

Figure 1. *Graphical solution of the Martin Shubik game.*

3. Some Conclusions Empirically Proven

3.1. Example "When Weakness Is Strength".

Let us theoretical examination of matter done before illustrate now with numerical example. Let us assume, that $p_A = 0{,}9$; $p_B = 0{,}6$; $p_C = 0{,}4$. Note, that probabilities satisfies requirements of table 1: $p_C = 0{,}4 > 0{,}0602$.

The surviving probabilities of two agents fighting game are showed in table 2. In table 3 the Nash situations of the sub-games and situations beside to the Nash equilibrium are exposed. In the table 4 the Nash equilibrium of Martin Shubik game is exposed.

Table 2. *Surviving probabilities in the two agents game if $p_A = 0{,}9$; $p_B = 0{,}6$; $p_C = 0{,}4$.*

$P_A(AB)$	0,93750	$P_A(AC)$	0,95745
$P_A(BA)$	0,37500	$P_A(CA)$	0,57447
$P_B(AB)$	0,06250	$P_B(BC)$	0,78947
$P_B(BA)$	0,62500	$P_B(CB)$	0,47368

$P_C(AC)$	0,04255	$P_C(BC)$	0,21053
$P_C(CA)$	0,42553	$P_C(CB)$	0,52632

Table 3. *The Nash equilibriums of the sub-games if p_A = 0,9; p_B = 0,6; p_C = 0,4.*

Sub-game (ABC)	**A**	**B**	**C**	**P_A(ABC)**	**P_B(ABC)**	**P_C(ABC)**
The Nash equilibrium	shot B	shot A	shot air	0,53856	0,02961	0,43183
	shot C	shot A	shot air	0,35156		
	shot B	shot C	shot air		0,00391	
	shot B	shot A	shot A			0,42820
Sub-game (ACB)	A	B	C	P_A(ACB)	P_B(ACB)	P_C(ACB)
The Nash equilibrium	shot B	shot A	shot air	0,53856	0,02961	0,43183
	shot C	shot A	shot air	0,351563		
	shot B	shot C	shot air		0,00391	
	shot B	shot A	shot A			0,42044
Sub-game (BAC)	A	B	C	P_A(BAC)	P_B(BAC)	P_C(BAC)
The Nash equilibrium	shot B	shot A	shot air	0,21543	0,29605	0,48852
	shot C	shot A	shot air	0,14063		
	shot B	shot C	shot air		0,03906	
	shot B	shot A	shot A			0,48396
Sub-game (BCA)	A	B	C	P_A(BCA)	P_B(BCA)	P_C(BCA)
The Nash equilibrium	shot B	shot A	shot air	0,21543	0,29605	0,48852
	shot C	shot A	shot air	0,14063		
	shot B	shot C	shot air		0,03906	
	shot B	shot A	shot A			0,45224
Sub-game (CAB)	A	B	C	P_A(CAB)	P_B(CAB)	P_C(CAB)
The Nash equilibrium	shot B	shot A	shot air	0,53856	0,02961	0,43183
	shot C	shot A	shot air	0,35156		
	shot B	shot C	shot air		0,00391	
	shot B	shot A	shot A			0,34113
Sub-game (CBA)	A	B	C	P_A(CBA)	P_B(CBA)	P_C(CBA)
The Nash equilibrium	shot B	shot A	shot air	0,21543	0,29605	0,48852
	shot C	shot A	shot air	0,14063		
	shot B	shot C	shot air		0,03906	
	shot B	shot A	shot A			0,37459

Table 4. *The Nash equilibrium of the Martin Shubik game if p_A = 0,9; p_B = 0,6; p_C = 0,4.*

The Martin Shubik game	A	B	C	P_A	P_B	P_C
The Nash equilibrium	shot B	shot A	shot air	0,37699	0,16283	0,46018
	shot C	shot A	shot air	0,24609		
	shot B	shot C	shot air		0,02148	
	shot B	shot A	shot A			0,41676

The examined example allows us to formulate following conclusions.

1. Strategy triple (A→B, B→A, C→O) in all six sub-games and, of course, also in Martin Shubik game leads to the Nash equilibrium.

2. In the Martin Shubik game in the Nash equilibrium situation the strategy "shot in the air" insures to Casis the highest surviving probability comparing other two agents. The weakest shooter has the biggest probability to survive! Exactly this result as universal conclusion Gintis (2009) sprightly discuss under the title "When Weakness Is Strength". By opinion of Herbert Gintis this result contradicts to the fundamental thesis of Charles Darvin "*survival of the fittest*".

Below in the example 2.2 "When Weakness Is Not Strength" the case when conclusion of Herbert Gintis does not hold is exposed.

3. Let us observe, that in some sub-games, for instance, in sub-game (ACB) the surviving probability of Casis is lower then surviving probability of Ansis. However if Ansis shots Basis and Basis shots Ansis, then in the each sub-game Casis shots in the air.

3.2. Example "When Weakness Is Not Strength".

Suppose $p_A = 0,9$; $p_B = 0,7$; $p_C = 0,2$.

Note, that probabilities satisfies requirements of table 1: $p_C = 0,2 > 0,0546$.

The surviving probabilities of two agents fighting game are showed in table 5. In table 6 the Nash situations of the sub-games and situations beside to the Nash equilibrium are exposed. In the table 7 the Nash equilibrium of Martin Shubik game is exposed.

Table 5. *Surviving probabilities in the two agents game if $p_A = 0,9$; $p_B = 0,7$; $p_C = 0,2$.*

$P_A(AB)$	0,92784	$P_A(AC)$	0,97826
$P_A(BA)$	0,27835	$P_A(CA)$	0,78261
$P_B(AB)$	0,07216	$P_B(BC)$	0,92105
$P_B(BA)$	0,72165	$P_B(CB)$	0,73684
$P_C(AC)$	0,02174	$P_C(BC)$	0,07895
$P_C(CA)$	0,21739	$P_C(CB)$	0,26316

Table 6. *The Nash equilibriums of the sub-games if $p_A = 0,9$; $p_B = 0,7$; $p_C = 0,2$.*

Sub-game (ABC)	A	B	C	$P_A(ABC)$	$P_B(ABC)$	$P_C(ABC)$
The Nash equilibrium	shot B	shot A	shot air	0,72613	0,05317	0,22069
	shot C	shot A	shot air	0,25826		
	shot B	shot C	shot air		0,00521	
	shot B	shot A	shot A			0,21982
Sub-game (ACB)	A	B	C	$P_A(ACB)$	$P_B(ACB)$	$P_C(ACB)$
The Nash equilibrium	shot B	shot A	shot air	0,72613	0,05317	0,22069
	shot C	shot A	shot air	0,258263		
	shot B	shot C	shot air		0,00521	
	shot B	shot A	shot A			0,21718
Sub-game (BAC)	A	B	C	$P_A(BAC)$	$P_B(BAC)$	$P_C(BAC)$
The Nash equilibrium	shot B	shot A	shot air	0,21784	0,53174	0,25042
	shot C	shot A	shot air	0,07748		
	shot B	shot C	shot air		0,05208	
	shot B	shot A	shot A			0,24936
Sub-game (BCA)	A	B	C	$P_A(BCA)$	$P_B(BCA)$	$P_C(BCA)$
The Nash equilibrium	shot B	shot A	shot air	0,21784	0,53174	0,25042
	shot C	shot A	shot air	0,07748		
	shot B	shot C	shot air		0,05208	
	shot B	shot A	shot A			0,24170
Sub-game (CAB)	A	B	C	$P_A(CAB)$	$P_B(CAB)$	$P_C(CAB)$
The Nash equilibrium	shot B	shot A	shot air	0,72613	0,05317	0,22069
	shot C	shot A	shot air	0,25826		
	shot B	shot C	shot air		0,00521	
	shot B	shot A	shot A			0,19165
Sub-game (CBA)	A	B	C	$P_A(CBA)$	$P_B(CBA)$	$P_C(CBA)$
The Nash equilibrium	shot B	shot A	shot air	0,21784	0,53174	0,25042
	shot C	shot A	shot air	0,07748		
	shot B	shot C	shot air		0,05208	
	shot B	shot A	shot A			0,21528

Table 7. *The Nash equilibrium of the Martin Shubik game if $p_A = 0,9$; $p_B = 0,7$; $p_C = 0,2$.*

The Martin Shubik game	A	B	C	P_A	P_B	P_C
The Nash equilibrium	shot B	shot A	shot air	0,47199	0,29246	0,23556
	shot C	shot A	shot air	0,16787		
	shot B	shot C	shot air		0,02864	
	shot B	shot A	shot A			0,22250

The examined example allows us to formulate following conclusions.

1. Strategy triple (A→B, B→A, C→O) in all six sub-games and, of course, also in Martin Shubik game leads to the Nash equilibrium.
2. In the Nash equilibrium situation of the Martin Shubik game surviving probability for Casis is the lowest to compare with two others agents. The weakest shooter has the lowest probability to survive! This result contradicts to the conclusion of Herbert Gintis (2009, p. 23).

3.3. The example for approving the Nash equilibrium condition.

Let us suppose, that $p_A = 0{,}7$; $p_B = 0{,}2$; $p_C = 0{,}1$.

The probabilities satisfies requirements of table 1: $p_C = 0{,}1 < 0{,}5894$.

The surviving probabilities of two agents fighting game are showed in table 8. Tables 9 and 10 show, that strategy triple (A→B, B→A, C→O) does not lead to the Nash equilibrium.

Table 8. *Surviving probabilities in the two agents game if* $p_A = 0{,}7$; $p_B = 0{,}2$; $p_C = 0{,}1$.

$P_A(AB)$	0,92105	$P_A(AC)$	0,95890
$P_A(BA)$	0,73684	$P_A(CA)$	0,86301
$P_B(AB)$	0,07895	$P_B(BC)$	0,71429
$P_B(BA)$	0,26316	$P_B(CB)$	0,64286
$P_C(AC)$	0,04110	$P_C(BC)$	0,28571
$P_C(CA)$	0,13699	$P_C(CB)$	0,35714

Table 9. *The situations of sub-games.*

Sub-game (ABC)	**A**	**B**	**C**	**$P_A(ABC)$**	**$P_B(ABC)$**	**$P_C(ABC)$**
It is no Nash equilibrium	shot B	shot A	shot air	0,79488	0,05075	0,15437
	shot C	shot A	shot air	0,67867		
	shot B	shot C	shot air		0,00623	
	shot B	shot A	shot A			0,15839
Sub-game (ACB)	A	B	C	$P_A(ACB)$	$P_B(ACB)$	$P_C(ACB)$
It is no Nash equilibrium	shot B	shot A	shot air	0,79488	0,05075	0,15437
	shot C	shot A	shot air	0,67867		
	shot B	shot C	shot air		0,00623	
	shot B	shot A	shot A			0,15784
Sub-game (BAC)	A	B	C	$P_A(BAC)$	$P_B(BAC)$	$P_C(BAC)$
It is no Nash equilibrium	shot B	shot A	shot air	0,63590	0,16917	0,19492
	shot C	shot A	shot air	0,54294		
	shot B	shot C	shot air		0,02078	
	shot B	shot A	shot A			0,19770
Sub-game (BCA)	A	B	C	$P_A(BCA)$	$P_B(BCA)$	$P_C(BCA)$
It is no Nash equilibrium	shot B	shot A	shot air	0,63590	0,16917	0,19492
	shot C	shot A	shot air	0,54294		
	shot B	shot C	shot air		0,02078	
	shot B	shot A	shot A			0,20833
Sub-game (CAB)	A	B	C	$P_A(CAB)$	$P_B(CAB)$	$P_C(CAB)$
It is no Nash equilibrium	shot B	shot A	shot air	0,79488	0,05075	0,15437
	shot C	shot A	shot air	0,67867		
	shot B	shot C	shot air		0,00623	
	shot B	shot A	shot A			0,17112
Sub-game (CBA)	A	B	C	$P_A(CBA)$	$P_B(CBA)$	$P_C(CBA)$
It is no Nash equilibrium	shot B	shot A	shot air	0,63590	0,16917	0,19492
	shot C	shot A	shot air	0,54294		
	shot B	shot C	shot air		0,02078	
	shot B	shot A	shot A			0,20650

Table 10. *The situations of the Martin Shubik game.*

The Martin Shubik game	A	B	C	P_A	P_B	P_C
It is no Nash equilibrium	shot B	shot A	shot air	0,71539	0,10996	0,17464

	shot C	shot A	shot air	0,61080		
	shot B	shot C	shot air		0,01350	
	shot B	shot A	shot A			0,18331

The examined example confirms characterization of the Nash equilibrium given into table 1 and in the figure 1 and allows to appraise critically the exposition of the theme "When Weakness Is Strength" in Gintis (2009).

4. Survives agent who is not the weakest but pretends to be weakest

As before we suppose that inequality $0 < p_C < p_B < p_A \leq 1$ holds.
Let us examine surviving game in circumstance when special kind of asymmetric information present.

4.1. Ansis is the strongest but pretends to be weakest.

Let us start with case when Ansis pretends to be weakest. Ansis assures Basis and Casis that probabilities satisfy the inequality $p_A < p_C < p_B$ and he − Ansis will shot in the air. In the reality Ansis is going to shot Basis.

If Basis and Casis believe that $p_A < p_C < p_B$ and A→O, then, as it was showed before, the strategies of Basis and Casis are B→C, C→B, which leads to the illusory conditional Nash equilibrium in the Basis and Casis imaginations. In the reality strategy triple (A→B, B→C, C→B) is realized, and that strategy triple guaranteed to Ansis the highest surviving probability.

Let us illustrate the case when Ansis is the strongest but pretends to be weakest with help of some examples.

Example 1. *"When pretence is strenght".*

Let us asume that in reality $p_A = 0{,}9$; $p_B = 0{,}6$; $p_C = 0{,}4$, but Ansis pretends to be weakest and Basis and Casis believe that $p_A < p_C < p_B$, namely, they believe that Ansis is the weakest and will shot in the air. The situations what arise in the six sub-games as result of strategy triple (A→B, B→C, C→B) are exposed in the table 11. Situation what arise in the Martin Shubik game is exposed in the table 12.

Table 11. *The situations of the sub-games if Ansis pretends to be the weakest.*

Sub-game (ABC)	B	C	A	$P_B(ABC)$	$P_C(ABC)$	$P_A(ABC)$
	shot C	shot B	shot B	0,03842	0,10115	0,86043
Sub-game (ACB)	B	C	A	$P_B(ACB)$	$P_C(ACB)$	$P_A(ACB)$
	shot C	shot B	shot B	0,03842	0,15766	0,80392
Sub-game (BAC)	B	C	A	$P_B(BAC)$	$P_C(BAC)$	$P_A(BAC)$
	shot C	shot B	shot B	0,02305	0,11161	0,86533
Sub-game (BCA)	B	C	A	$P_B(BCA)$	$P_C(BCA)$	$P_A(BCA)$
	shot C	shot B	shot B	0,00231	0,25288	0,74482
Sub-game (CAB)	B	C	A	$P_B(CAB)$	$P_C(CAB)$	$P_A(CAB)$
	shot C	shot B	shot B	0,00384	0,39309	0,60306
Sub-game (CBA)	B	C	A	$P_B(CBA)$	$P_C(CBA)$	$P_A(CBA)$
	shot C	shot B	shot B	0,00231	0,39414	0,60355

Table 12. *The situations of the Martin Shubik game if Ansis pretends to be the weakest.*

Martin Shubik game	B	C	A	P_B	P_C	P_A
	shot C	shot B	shot B	0,01806	0,23509	0,74685

Let us compare this example with example "When Weakness Is Strength" in the section 2.1.
We have $p_A = 0{,}9$; $p_B = 0{,}6$; $p_C = 0{,}4$ in both examples. In the tables 2 and 3 are exposed surviving probabilities of all agents in case when all players now real proportion. In the tables 11 and 12 are exposed surviving probabilities of all agents in case when the strongest player pretends to be weakest, but two other agents do not know true. In the first case in the Martin Shubik game $P_A = 0{,}38$, in the second case $P_A = 0{,}75$. It is advantageous do not expose ones power! Let us note, that in the second case very low surviving probability has Basis because of not only Ansis shots to him, but also Casis shots Basis.

Example 2. *"When pretence is strenght".*

Let us asume that in reality the predominance of Ansis is smaller: $p_A = 0,7$; $p_B = 0,6$; $p_C = 0,5$. Ansis pretends to be weakest and Basis and Casis believe that Ansis is the weakest and will shot in the air. The situations what arise in the Martin Shubik game as result of strategy triple (A→B, B→C, C→B) are exposed in the table 13. Obviously, the pretence of Ansis helps him to survive.

Table 13. *The situation of the Martin Shubik game if Ansis pretends to be the weakest.*

Martin Shubik game	B	C	A	P_B	P_C	P_A
	shot C	shot B	shot B	0,06746	0,28692	0,64562

4.2. The second strongest player Basis pretends to be weakest.

Now we are going to demonstrate that pretence strategy provides the highest surviving probability also for the second strongest player − Basis.

For instance, let us examine the case, whe in the reality $p_A = 0,7$; $p_B = 0,6$; $p_C = 0,5$. Basis pretends to be weakest and Ansis and Casis believe that $p_A < p_C < p_B$, namely, they believe that Basis is the weakest and will shot in the air. The situations what arise as result of strategy triple (A→C, B→A, C→A) in the six sub-games are exposed in the table 14. The situation what arises in the Martin Shubik game is exposed in the table 15.

Table 14. *The situations of the sub-games if Basis pretends to be the weakest.*

Sub-game (ABC)	A	C	B	$P_A(ABC)$	$P_C(ABC)$	$P_B(ABC)$
	shot C	shot A	shot A	0,23694	0,09973	0,66332
Sub-game (ACB)	A	C	B	$P_A(ACB)$	$P_C(ACB)$	$P_B(ACB)$
	shot C	shot A	shot A	0,23694	0,13564	0,62742
Sub-game (BAC)	A	C	B	$P_A(BAC)$	$P_C(BAC)$	$P_B(BAC)$
	shot C	shot A	shot A	0,11847	0,19282	0,68871
Sub-game (BCA)	A	C	B	$P_A(BCA)$	$P_C(BCA)$	$P_B(BCA)$
	shot C	shot A	shot A	0,04739	0,33245	0,62016
Sub-game (CAB)	A	C	B	$P_A(CAB)$	$P_C(CAB)$	$P_B(CAB)$
	shot C	shot A	shot A	0,09478	0,41489	0,49033
Sub-game (CBA)	A	C	B	$P_A(CBA)$	$P_C(CBA)$	$P_B(CBA)$
	shot C	shot A	shot A	0,04739	0,45213	0,50048

Table 15. *The situations of the Martin Shubik game if Basis pretends to be the weakest.*

Martin Shubik game	A	C	B	P_A	P_C	P_B
	shot C	shot A	shot A	0,13032	0,27128	0,59840

Numerical experiments performed show us that the surviving probability which Basis gets do to pretending is the largest to compare with another two surviving probabilities. In some sub-games surviving probability of pretenders is even larger then pretenders probability to hit.

5. Conclusions

The author is convinced of cardinal role of game theory in the investigation of individuals' strategy interaction what is fundament for understanding absolutely all social and economic processes. In the circumstance of global confidence crisis the investigation of asymmetric information role in individuals' interactions seems for us especially actual. In present paper the surviving game of three agents, what essence is the competition fight, is investigated. It is established that the shape of Nash equilibrium in a specific way depends of surviving probabilities of the agents. In the paper the limitation of some statements of Herbert Gintis empirically is proved. By opinion of author, the most valuable result of this research is the solution of Martin Shubik surviving game exposed in geometrical form. It is recognized that statement of Martin Shubik "survives the weakest" in circumstances of asymmetric information transforms in statement "survives agent who is not the weakest but pretends to be weakest". My recommendation is to include the surviving game as topic in the game theory course taught in academic program of economics.

6. References

Binmore, K., 2007. *Game Theory. A Very Short Introduction.* Oxford: Oxford University Press

Gintis H., 2009. *Game Theory Evolving: A Problem Centered Introduction to Modelling Strategic Interaction – 2nd Edition.* Princeton, NJ: Princeton University Press

Jaunzems, A., 2008. *Mikroekonomika. Vidēja teorētiskā līmeņa kurss - 1. daļa*., Ventspils Augstskola, "LiePA"

Jaunzems, A., 2009a. *Mikroekonomika. Vidēja teorētiskā līmeņa kurss - 2. daļa*., Drukātava

Jaunzems, A., 2009b. *Risku analīze un vadīšana*. Drukātava

Jaunzems, A., 2009c. Trimatricu spēles: interpretācijas un pielietojumi sociālekonomisko situāciju stabilitātes analīzē, in Medveckis A. (editor), *Sabiedrība un kultūra*. Rakstu krājums, XI / Sastād, Liepāja: LiePA, pp. 521-534

Майкл, Р.Б., 1999. *Управленческая экономика и стратегия бизнеса*. Москва: Юнити, 743 стр.

Shubik, M., 1954. *Readings in Game Theory and Political Behavior*, New York: Doubleday

Байе, М.Р., 1999. *Управленческая экономика и стратегия бизнеса.* Москва: Юнити,